Meramec College Algebra Chapter 6 TEST 5 Solution Summer 2010

NAME: ______________________________ Score______________________________/100

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SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION Course Average___________

1. **T** F A matrix is a rectangular array of numbers.
2. **T** F Matrix addition is commutative.
3. T **F** Matrix multiplication is commutative.
4. T **F** The inner product of two matrices is a matrix.
5. **T** F If A and B are square matrices with the same dimension, then det(AB) = det(A)det(B).
6. **T** F The rule for the det function is a recursive rule.
7. **T** F If A is a square matrix and det(A) = 0, then A does not have an inverse.
8. T **F** If two matrices have the same order, their product is defined.
9. T **F** Only square matrices can be multiplied.
10. **T** F Every square matrix has a determinant.
11. T **F** Division of matrices is defined as a multiplication.
12. If a matrix has m rows and n columns, the order of the matrix is $m \times n$.
13. If a matrix has only one row, it is called a **row** matrix.
14. If a matrix has only one column, it is called a **column** matrix.
15. Two matrices and are equal if they have the same **order** and their corresponding **entries** are equal.
16. The **identity** matrix of order n is the matrix whose main diagonal entries are 1 and all other entries are 0.
17. The determinant is a function whose domain is the set of **square** matrices and whose range is the **real** numbers.
18. The matrix consisting of the coefficients (in the same order) of a system of equations is called the **coefficient** matrix.
19. Write the 3×3 identity matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
20. The entry in the 34 position of the matrix AB is the inner product of row **3** in matrix A and column **4** in matrix B.
21. Write the coefficient matrix of the following system of equations: $\begin{cases} -2x + 2y - 4z = 1 \\ 2x - 5y - z = 6 \\ 4x + 2y - 3z = 5 \end{cases}$ $\begin{bmatrix} -2 & 2 & -4 \\ 2 & -5 & -1 \\ 4 & 2 & -3 \end{bmatrix}$
22. Write a matrix equation which is equivalent to the system $\begin{cases} 3x + 7y - 4z = 5 \\ 2x - 3y - 5z = 3 \\ -2x + 3y - 6z = 0 \end{cases}$

$$\begin{bmatrix} 3 & 7 & -4 \\ 2 & -3 & -5 \\ -2 & 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$$

23. Complete the statements of the **Elementary Row Operations:**

a. Interchange two **rows**.

b. **Multiply** a row by a non-zero **constant** and **replace** that row with the **product**.

c. Add a **multiple** of a row to another **row** and replace **one** but not both of the **both** with that **sum**.

24. Calculate $\det\left(\begin{bmatrix} 5 & 1 \\ 6 & 2 \end{bmatrix}\right) = \mathbf{(5)(2) - (6)(1) = 4}$

25. Calculate the inner product $\begin{bmatrix} 2 & -2 & 3 \end{bmatrix}\begin{bmatrix} -4 \\ -1 \\ 2 \end{bmatrix} = (2)(-4) + (-2)(-1) + (3)(2) = -8 + 2 + 6 = 0$

26. Perform the addition $\begin{bmatrix} 2 & 5 & 2 \\ -1 & 0 & 6 \\ 4 & -2 & 1 \end{bmatrix} + \begin{bmatrix} -8 & 5 & -1 \\ -5 & 6 & -4 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -6 & 10 & 1 \\ -6 & 6 & 2 \\ 5 & -1 & 1 \end{bmatrix}$

27. Calculate

$$\det\left(\begin{bmatrix} 3 & 6 & 4 \\ 1 & 0 & 2 \\ -1 & -6 & 5 \end{bmatrix}\right) = (1)(-1)^{2+1}\det\left(\begin{bmatrix} 6 & 4 \\ -6 & 5 \end{bmatrix}\right) + (2)(-1)^{2+3}\det\left(\begin{bmatrix} 3 & 6 \\ -1 & -6 \end{bmatrix}\right)$$

$$= (1)(-1)\left[(6)(5) - (-6)(4)\right] + (2)(-1)\left[(3)(-6) - (-1)(6)\right]$$

$$= (-1)\left[30 + 24\right] + (-2)\left[-18 + 6\right]$$

$$= -\left[54\right] - 2\left[-12\right] = -54 + 24 = -30$$

28. The inverse of $A = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$ **is the matrix** $\mathbf{A^{-1}} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

Use this information to solve the system $\begin{cases} -2x - 3y + z = 2 \\ -3x - 3y + z = 0 \\ -2x - 4y + z = 3 \end{cases}$

The system is equivalent to the matrix equation

$\begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$ **and since the inverse of** $\begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$ **is** $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$ **we conclude**

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ **from which it follows that x = 2, y = 0, and z = 3.**

Therefore the solution is the ordered triple (2, -1, 3).

Note the solution set is the set containing the solution so it is {(2, -1, 3)}.

29. Supply the missing entries by performing the indicated elementary row operation.

$$\begin{bmatrix} 2 & -2 & 1 & 0 \\ 3 & -3 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{3}R_2 \longrightarrow R_2} \begin{bmatrix} \boxed{2} & \boxed{-2} & \boxed{1} & \boxed{0} \\ \boxed{1} & \boxed{-1} & \boxed{0} & \boxed{\frac{1}{3}} \end{bmatrix}$$

30. Supply the missing entries by performing the indicated elementary row operation

$$\begin{bmatrix} -2 & 3 & \frac{1}{5} & 0 \\ 6 & -3 & 0 & 1 \end{bmatrix} \xrightarrow{3R_1 + R_2 \longrightarrow R_2} \begin{bmatrix} \boxed{-2} & \boxed{3} & \boxed{\frac{1}{5}} & \boxed{0} \\ \boxed{0} & \boxed{6} & \boxed{\frac{3}{5}} & \boxed{1} \end{bmatrix}$$

31. Perform the multiplication:

$$\begin{bmatrix} 2 & 3 & -4 \\ -1 & -3 & 3 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 26 & -11 \\ -21 & 8 \end{bmatrix}$$

32. You do not need to do any computations. Simply fill in the blanks to describe the process for finding the inverse of a matrix.

To find the inverse of the matrix $A = \begin{bmatrix} 5 & 7 & 4 \\ 3 & -1 & 3 \\ 6 & 7 & 5 \end{bmatrix}$

Begin by adjoining the **identity** matrix to obtain the matrix $\begin{bmatrix} 5 & 7 & 4 & 1 & 0 & 0 \\ 3 & -1 & 3 & 0 & 1 & 0 \\ 6 & 7 & 5 & 0 & 0 & 1 \end{bmatrix}$ with order 3×6

The next step is to get a **1** in the **11** position.

Then use that **1** to get **0** everywhere else in the **first column**

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 7/5 & 4/5 & 1/5 & 0 & 0 \\ 0 & -26/5 & 3/5 & -3/5 & 1 & 0 \\ 0 & -7/5 & 1/5 & -6/5 & 0 & 1 \end{bmatrix}$

The next step is to get a **1** in the **22** position.

Then use that **1** to get **0** everywhere else in the **second column**

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 0 & 25/26 & 1/26 & 7/26 & 0 \\ 0 & 1 & -3/26 & 3/26 & -5/26 & 0 \\ 0 & 0 & 1/26 & -27/26 & -7/26 & 1 \end{bmatrix}$

The next step is to get a **1** in the **33** position.

Then use that **1** to get **0** everywhere else in the **third column**

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 0 & 0 & 26 & 7 & -25 \\ 0 & 1 & 0 & -3 & -1 & 3 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{bmatrix}$

The inverse of A is matrix $A^{-1} = \begin{bmatrix} 26 & 7 & -25 \\ -3 & -1 & 3 \\ -27 & -7 & 26 \end{bmatrix}$

Which has order 3×3 __________