# Solution MERAMEC College Algebra Problem Sheet 4

**Name**________________________________ **Score** ________/6

Last name, First Name Please Print Clearly

**This assignment consists of four pages. All four pages are to remain stapled and turned in together.**

**Your submitted work should NOT be your first draft. Work the problem and then NEATLY copy your best work onto the three assignment pages. Write on the lines provided!**

**Your work MUST be modeled after the Sample Problem presented on this first page. That includes all the words. Remember it is the logic which SOLVES the problem NOT arbitrary/random/trial-and-error arithmetic computations.**

**DO NOT use a calculator. DO NOT use decimal approximations.**

**ABOVE ALL -- Think about what you are doing!**

**Sample Problem:** **Find the rule of the linear function for which** $f(-2)=\frac{3}{5}$ **and** $f\left(\frac{1}{2}\right)=-1$.

**Analysis:** **Since the desired function is linear it's rule has the form f(x) = mx + b and its graph is a line.**

**Since** $f(-2)=\frac{3}{5}$, **the point** $\left(-2,\frac{3}{5}\right)$ **is on the graph.**

**Since** $f\left(\frac{1}{2}\right)=-1$, **the point** $\left(\frac{1}{2},-1\right)$ **is on the graph.**

**The slope of the line through the two points is** $m=\frac{y_1-y_2}{x_1-x_2}=\frac{-1-\frac{3}{5}}{\frac{1}{2}-(-2)}=\frac{-\frac{8}{5}}{\frac{5}{2}}=\left(-\frac{8}{5}\right)\left(\frac{2}{5}\right)=-\frac{16}{25}$.

**Therefore the form of the rule for the desired function is** $f(x)=-\frac{16}{25}x+b$

**Since** $f\left(\frac{1}{2}\right)=-1$, **we have** $-1=f\left(\frac{1}{2}\right)=\left(-\frac{16}{25}\right)\left(\frac{1}{2}\right)+b=\left(-\frac{8}{25}\right)+b$

**The y-intercept b may be determined by solving this equation.**

$$-1=\left(-\frac{8}{25}\right)+b$$

$$b=-1+\frac{8}{25}=-\frac{17}{25}$$

**Finally the rule for the desired function is** $f(x)=-\frac{16}{25}x-\frac{17}{25}$

**Problem:** **Find the rule of the linear function for which** $f(3)=\frac{7}{3}$ **and** $f(1)=5$.

**Analysis: Since the desired function is linear it's rule has the form f(x) = mx + b and its graph is a line.**

**Since** $f(3)=\frac{7}{3}$**, the point** $\left(3,\ \frac{7}{3}\right)$ **is on the graph.**

**Since f(1) = 5, the point (1, 5) is on the graph.**
**The slope of the line through the two points is**

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{\frac{7}{3}-5}{3-1}=\frac{\frac{7-15}{3}}{2}=\frac{-\frac{8}{3}}{\frac{2}{1}}=\left(-\frac{8}{3}\right)\left(\frac{1}{2}\right)=-\frac{4}{3}$$

**Therefore the form of the rule for the desired function is** $f(x)=-\frac{4}{3}x+b$

**Since f(1) = 5, we have** $5=f(1)=-\frac{4}{3}(1)+b=-\frac{4}{3}+b$

**The y-intercept b may be determined by solving this equation.**

$$5=-\frac{4}{3}+b$$

$$b=5+\frac{4}{3}=\frac{19}{3}$$

**Finally the rule for the desired function is** $f(x)=-\frac{4}{3}x+\frac{19}{3}$

**ALTERNATE options are shown in blue**
**Since the desired function is linear it's rule has the form f(x) = mx + b and its graph is a line.**

**Since** $f(3)=\frac{7}{3}$**, the point** $\left(3,\ \frac{7}{3}\right)$ **is on the graph.**

**Since f(1) = 5, the point (1, 5) is on the graph.**
**The slope of the line through the two points is**

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{5-\frac{7}{3}}{1-3}=\frac{\frac{15-7}{3}}{-2}=\frac{\frac{8}{3}}{-\frac{2}{1}}=\left(\frac{8}{3}\right)\left(-\frac{1}{2}\right)=-\frac{4}{3}$$

**Therefore the form of the rule for the desired function is** $f(x)=-\frac{4}{3}x+b$

**Since** $f(3)=\frac{7}{3}$**, we have** $\frac{7}{3}=f(3)=-\frac{4}{3}(3)+b=-4+b$

**The y-intercept b may be determined by solving this equation.**

$$\frac{7}{3}=-4+b$$

$$b=\frac{7}{3}+4=\frac{7+12}{3}=\frac{19}{3}$$

**Finally the rule for the desired function is** $f(x)=-\frac{4}{3}x+\frac{19}{3}$

**Problem:** **Find the rule of the linear function for which** $f(3)=\frac{7}{3}$ **and whose graph contains the point (5, -2) .**

**Analysis: Since the desired function is linear it's rule has the form f(x) = mx + b and its graph is a line.**

**Since** $f(3)=\frac{7}{3}$**, the point** $\left(3,\ \frac{7}{3}\right)$ **is on the graph. Note the point (5, -2) is also on the graph.**

**The slope of the line through the two points is**

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{-2-\frac{7}{3}}{5-3}=\frac{\frac{-6-7}{3}}{2}=\left(-\frac{13}{3}\right)\left(\frac{1}{2}\right)=-\frac{13}{6}$$

**Therefore the form of the rule for the desired function is** $f(x)=-\frac{13}{6}x+b$

**Since** $f(3)=\frac{7}{3}$**, we have** $\frac{7}{3}=f(3)=-\frac{13}{6}(3)+b=-\frac{13}{2}+b$

**The y-intercept b may be determined by solving this equation.**

$$\frac{7}{3}=-\frac{13}{2}+b$$

$$b=\frac{7}{3}+\frac{13}{2}=\frac{14}{6}+\frac{39}{6}=\frac{53}{6}$$

**Finally the rule for the desired function is** $f(x)=-\frac{13}{6}x+\frac{53}{6}$

**ALTERNATE options are shown in blue**

**Since the desired function is linear it's rule has the form f(x) = mx + b and its graph is a line.**

**Since** $f(3)=\frac{7}{3}$**, the point** $\left(3,\ \frac{7}{3}\right)$ **is on the graph. Note the point (5, -2) is also on the graph.**

**The slope of the line through the two points is**

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{\frac{7}{3}-(-2)}{3-5}=\frac{\frac{7+6}{3}}{-2}=-\left(\frac{13}{3}\right)\left(\frac{1}{2}\right)=-\frac{13}{6}$$

**Therefore the form of the rule for the desired function is** $f(x)=-\frac{13}{6}x+b$

**Since(5, -2) is on the graph, f(5) = -2 and then we have** $-2=f(5)=-\frac{13}{6}(5)+b=-\frac{65}{6}+b$

**The y-intercept b may be determined by solving this equation.**

$$-2=-\frac{65}{6}+b$$

$$b=-2+\frac{65}{6}=\frac{-12+65}{6}=\frac{53}{6}$$

**Finally the rule for the desired function is** $f(x)=-\frac{13}{6}x+\frac{53}{6}$

**Problem:** **Find the rule of the linear function for which** $f(3)=\sqrt{5}$ **and** $f\left(\frac{2}{3}\right)=\frac{5}{4}$.

**Analysis:** **Since the desired function is linear it's rule has the form f(x) = mx + b and its graph is a line.**

**Since** $f(3)=\sqrt{5}$**, the point** $\left(3,\ \sqrt{5}\right)$ **is on the graph.**

**Since** $f\left(\frac{2}{3}\right)=\frac{5}{4}$**, the point** $\left(\frac{2}{3},\frac{5}{4}\right)$ **is on the graph.**

**The slope of the line through the two points is**

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{\frac{5}{4}-\sqrt{5}}{\frac{2}{3}-3}=\frac{\frac{5-4\sqrt{5}}{4}}{\frac{2-9}{3}}=\left(\frac{5-4\sqrt{5}}{4}\right)\left(-\frac{3}{7}\right)=\frac{-15+12\sqrt{5}}{28}$$

**Therefore the form of the rule for the desired function is** $f(x)=\left(\frac{-15+12\sqrt{5}}{28}\right)x+b$

**Since** $f\left(\frac{2}{3}\right)=\frac{5}{4}$**, we have** $\frac{5}{4}=f\left(\frac{2}{3}\right)=\left(\frac{-15+12\sqrt{5}}{28}\right)\left(\frac{2}{3}\right)+b=\left(\frac{-5+4\sqrt{5}}{14}\right)\left(\frac{\cancel{3}}{\cancel{2}}\right)\left(\frac{\cancel{2}}{\cancel{3}}\right)+b=\left(\frac{-5+4\sqrt{5}}{14}\right)+b$

**The y-intercept b may be determined by solving this equation.**

$$\frac{5}{4}=\left(\frac{-5+4\sqrt{5}}{14}\right)+b$$

$$b=\frac{5}{4}-\left(\frac{-5+4\sqrt{5}}{14}\right)=\frac{35}{28}-\left(\frac{-10+8\sqrt{5}}{28}\right)=\frac{35+10-8\sqrt{5}}{28}=\frac{45-8\sqrt{5}}{28}$$

**Finally the rule for the desired function is** $f(x)=\left(\frac{15-12\sqrt{5}}{28}\right)x+\frac{45-8\sqrt{5}}{28}$

**Some Alternate Computations**

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{\sqrt{5}-\frac{5}{4}}{3-\frac{2}{3}}=\frac{\frac{4\sqrt{5}-5}{4}}{\frac{9-2}{3}}=\left(\frac{4\sqrt{5}-5}{4}\right)\left(\frac{3}{7}\right)=\frac{-15+12\sqrt{5}}{28}$$

$$\sqrt{5}=f(3)=\left(\frac{-15+12\sqrt{5}}{28}\right)(3)+b=\frac{-45+36\sqrt{5}}{28}+b$$

$$b=\sqrt{5}-\left(\frac{-45+36\sqrt{5}}{28}\right)=\frac{28\sqrt{5}+45-36\sqrt{5}}{28}=\frac{45+(28-36)\sqrt{5}}{28}=\frac{45-8\sqrt{5}}{28}$$