Meramec College Algebra TEST 1a Spring 2006

NAME: ______________________________ Score______________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct.

Problems 1 – 20 are each worth 1 point.

1. **T** F If an expression is added to both sides of an equality, the result is an equality which is equivalent to the first equality.

2. T **F** If both sides of an equality are multiplied by a negative real number, the result is an equality which is equivalent to the original equality.

3. T **F** If both sides of an equation are squared, the resulting equation is equivalent to the original equation.

4. **T** F Every integer is a real number.

5. T **F** If the solution set of an equation is {3, 8, 11} and the solution set of another equation is {3, 11, 15}, then the two equations are equivalent.

6. T **F** x + 5 is a term.

7. **T** F Every irrational number is a complex number.

8. T **F** Every real number is a rational number.

9. T **F** Every quadratic equation in one variable has two real solutions.

10. T **F** If a + bi and c + di are two complex numbers then their sum is ac + bdi

11. T **F** The product of a complex number and its norm is 1

Fill in each of the blanks to make the statements true.

12. An **equation** is a mathematical statement which contains an = sign.

13. A number (or numbers) that makes an equation true when substituted for the variable (or variables) is called a **solution** of the equation.

14. Two equations are **equivalent** if they have the same solution sets

15. A simplest equation is an equation which has a single **variable** on one side of the equal sign and a single **number** on the other side

16. A linear equation in one variable is an equation that can be written in the form **ax + b = 0** where a and b are real numbers with a not zero.

17. If k is a positive real number then the principal square root of its opposite -k is defined by

$\sqrt{-k} = i\sqrt{k}$.

18. The graph of a quadratic equation in two variables is a **parabola** which opens **up** if **the leading coefficient is positive** and opens **down** if **the leading coefficient is negative**

19. A quadratic equation in one variable x is an equation which may be written in the form

$ax^2 + bx + c = 0$ where a, b, and c are real numbers and a is not zero.

20. The equation $x^2 + y^2 = r^2$ is the equation of the **circle** with **center** at the origin and **radius** r.

Problems 21 – 30 are each worth 3 points. Most of these require very little of no work.

21. Suppose A is the solution set for $x^3 + 2x + 17 = 43$ and

B is the solution set for $(x + 1)(x^3 + 2x + 17) = (x + 1)(43)$

What is the relationship between A and B?

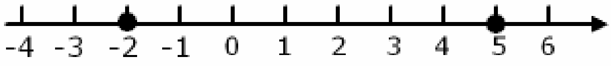
$A \subseteq B$

When an equation is multiplied by an expression containing a variable, the solution set of the resulting equation contains the solution set of the original equation.

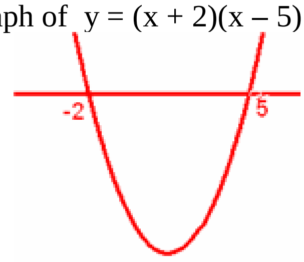
22. Write the equation of the circle with center (–4, 5) and radius 2.

$(x + 4)^2 + (y – 5)^2 = 2^2$

23. Sketch the graph of the equation $(x + 2)(x – 5) = 0$.



24. Sketch the graph of $y = (x + 2)(x – 5)$



25. Show that 1 is a solution of the equation $x^5 + 3x^4 – 2x^3 + 6x^2 – 12x + 4 = 0$.

$1^5 + 3(1^4) – 2(1^3) + 6(1^2) + 4 = 1 + 3 – 2 + 6 – 12 + 4 = 0$ is TRUE.

Therefore 1 is a solution of the equation.

26. What is the y-intercept of the graph of the equation $y = x^5 + 3x^4 – 2x^3 + 6x^2 – 12x + 4$?

To find the y-intercept of an equation, set x = 0 and solve for y. In this case if x = 0, y = -7. The y-intercept is -7.

27. Write the complex number $4 + \sqrt{-5}$ in standard form a + bi. $4 + \sqrt{-5} = 4 + i\sqrt{5}$

28. Write the norm of the complex number 5 + 2i. **The norm is $5^2 + 2^2 = 29$.**

29. Write the multiplicative inverse of the complex number 2 – 5i.

The multiplicative inverse of 2 – 5i is its conjugate divided by it norm $\frac{2+5i}{2^2+5^5} = \frac{2+5i}{29}$

30. Write the set {x| 2< x < 4} in interval notation and sketch its graph.

{x| 2< x < 4} = (2, 4)

Problems 31 – 40 are each worth 5 points.

31. 52 is 14% of what number.

Percentage = (percent)(base)

52 = (0.14)(base)

Base = $\frac{52}{0.14}$ **= 371.43 (accurate to two decimal places)**

32. Write the equation $\sqrt{5}x+3y=17x+9$ in slope-intercept form.

Begin with the original equation

$\sqrt{5}x+3y=17x+9$ **Add** $-\sqrt{5}x$ **to both sides**

$3y=17x-\sqrt{5}x+9=\left(17-\sqrt{5}\right)x+9$ **Multiply both sides by** $\frac{1}{3}$

$y=\left(\frac{17-\sqrt{5}}{3}\right)x+3$ **This is the desired slope-intercept form.**

33. Find the solution set for $\boldsymbol{x}=\sqrt{-5\boldsymbol{x}-6}$

$\boldsymbol{x}=\sqrt{-5\boldsymbol{x}-6}$	**Square both sites**
$\mathbf{x^2 = -5x - 6}$	**Add -5x – 6 to both sides**
$\mathbf{x^2 + 5x + 6 = 0}$	**Factor**
(x + 3)(x + 2) = 0	**Use the Zero Factor Property**
x + 3 = 0 OR x + 2 = 0	**Solve the two equations** .

The solution set for $\mathbf{x^2 = -5x - 6}$ **is {-2, -3}**

Test -2: $-2=\sqrt{-5(-2)-6}$ **is false because the principal square root cannot be negative.**

Similarly –3 cannot equal a positive square root.

Therefore neither – 2 nor – 3 are solutions

The solution set for the original equation $x=\sqrt{-5x-6}$ **is the null set** $\varnothing$

34. Use the Quadratic Formula to find the solution set for $2x^2 + x - 1 = 0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1^2-(4)(2)(1)}}{(2)(2)}=\frac{-1\pm\sqrt{9}}{4}=\frac{-1\pm 3}{4}$$

The solution set for $2x^2 + x - 1 = 0$ is $\left\{-1,\frac{1}{2}\right\}$

35. Solve the equation $\boldsymbol{C}=\frac{5}{9}\left(\boldsymbol{F}-32\right)$ for F.

$\boldsymbol{C}=\frac{5}{9}\left(\boldsymbol{F}-32\right)$ **multiply by** $\mathbf{\frac{9}{5}}$

$\frac{9}{5}\boldsymbol{C}=\boldsymbol{F}-32$ **add 32 to both sides**

$\frac{9}{5}\boldsymbol{C}+32=\boldsymbol{F}$

36. Compute the sum (3 – 7i) + (3 + 4i) **= (3 + 3) + (-7 + 4)i =6 - 3i**

37. Compute the product (3 + 2i)(7 – 5i) **= (3)(7) +(3)(-5i) + (2i)(7) + (2i)(-5i)**
= 21 - 15i +14i +10 = 31 – i

38. Compute the quotient (7 – 5i) ÷(3 + 2i)

$$(7-5i)\div(3+2i)=(7-5i)\left(\frac{3-2i}{3^2+2^2}\right)=\frac{(7-5i)(3-2i)}{13}$$

$$=\frac{1}{13}\left(21-14i-15i+10i^2\right)=\frac{1}{13}(11-29i)$$

39. When solving the equation $5x^3 - 4x^2 + x = 5x^3 + 6$ we might add $-5x^3$ to both sides of the equation to obtain $-4x^2 + x = 6$. What is the relationship between the original equation and this second equation? State a property which justifies your answer.
The two equations are equivalent because when any expression is added to both sides of an equation the resulting equation is equivalent to the original equation. In this case the expression -5x was added to both sides of the original equation.

40. A can manufacturer has a contract to make cylindrical cans with a radius of 2 inches and a volume of 15 cubic inches. What should be the height of the cans?

$$V=\pi r^2 h \quad \text{so } h=\frac{V}{\pi r^2}=\frac{15}{\pi 2^2}=\frac{15}{4\pi}$$