Meramec College Algebra TEST 1 C Solution Spring 2006

NAME: _____Score_____/100
Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct.
Problems 1 – 20 are each worth 1 point.

1. **T** F If an expression is added to both sides of an equality, the result is an equality which is equivalent to the first equality.

2. **T** F The x-intercepts of a graph are found by setting y = 0 and solving for x.

3. T **F** If the same operation is performed on both sides of an equation the resulting equation is equivalent to the original equation.

4. **T** F Every natural number is a rational number.

5. T **F** If the solution set of an equation is {3, 8, 11} and the solution set of another equation is {3, 11,}, then the two equations are equivalent.

6. T **F** The leading term of $3x^7 + 5x^3 – 2x^9$ is $3x^7$.

7. **T** F Every irrational number is a complex number.

8. **T** F $\{ x \mid 2 \le x < 4\} = [2, 4)$.

9. T **F** The complex component of the complex number 3 + 5i is 5i.

10. **T** F If a + bi and c + di are two complex numbers then their sum is (a + c) + (b + d)i

11. T **F** The product of a complex number and its conjugate is 1

Fill in each of the blanks to make the statements true.

12. An equation is a mathematical statement which contains an **= sign**.

13. A number (or numbers) that makes an equation true when substituted for the variable (or variables) is called a **solution** of the equation.

14. Two equations are equivalent if they have the same **solution set**.

15. A simplest equation is an equation which has a single **variable** on one side of the equal sign and a single **number** on the other side

16. The numerical factor of a term is called its **coefficient**.

17. The principle square root of -5 is written as $\sqrt{-5}$ and is equal to $i\sqrt{5}$.

18. Describe the graph of the equation y = (x + 3)(x + 5). Do not sketch a graph. _____

The graph is a parabola which opens up and has x-intercepts –5 and –3.

19. The equation y = 3x – 7 is a **linear** equation in **two** variables.

20. The equation $\left(x-\frac{3}{5}\right)^2+\left(y+\sqrt{5}\right)^2=11$ is the equation of the **circle** with center at $\left(\frac{3}{5},-\sqrt{5}\right)$ and radius $\sqrt{11}$.

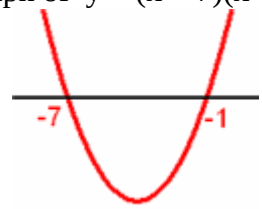
Problems 21 – 30 are each worth 3 points. Most of these require very little or no work.

21. Suppose A is the solution set for $\sqrt{x^3+5x^2-7}=2$ and
B is the solution set for $x^3+5x^2-7=4$
What is the relationship between A and B? **$A \subseteq B$**

22. Write the equation of the circle with center (6, 3) and radius $\sqrt{6}$. **$(x-6)^2+(y-3)^2=6$**

23. Consider the two equations $\frac{x+1}{x-2}=\frac{3x-5}{(x+1)(x-4)}$ and $(x+1)(x-4)(x+1)=(3x-5)(x-2)$. Are they equivalent? Give a reason for your answer. **Because both sides of the equation were multiplied by the same variable expression we know the solution set of the original equation is contained in the solution set of the second equation. We do not know if the two equations are equivalent because we do not know whether their solutions sets are equal.**

24. Sketch the graph of y = (x + 7)(x + 1)



25. State the Quadratic Formula. **The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by**

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

26. Write the interval (-2, 5] in set builder notation? **$(-2, 5] = \{ x \mid -2 < x \leq 5 \}$**

27. Write the complex number $-5+\sqrt{-8}$ in standard form a + bi. $-5+\sqrt{-8}=-5+i\sqrt{8}$
This can be further simplified $-5+i\sqrt{8}=-5+i2\sqrt{2}$

28. Write the norm of the complex number 5 - 3i. **The norm of 5 – 3i is $5^2 + 3^2 = 34$**

29. Write the multiplicative inverse of the complex number $5+3i$.
The multiplicative inverse of 5 + 3i is $\frac{5-3i}{34}$ which can also be written as $\frac{5}{34}-\frac{3}{34}i$

30. Write the set {x| 2< x < 4} in interval notation. **{x| 2< x < 4} = (2, 4)**

Problems 31 – 40 are each worth 5 points.

31. What is 23% of 0.46.

Percentage = (percent)(base)

Percentage = (0.23)(0.46) = 0.1058

32. Write the equation $\sqrt{3}x+\sqrt{5}y=\frac{4}{3}x+9$ in slope-intercept form.

Begin with the original equation

$\sqrt{3}x+\sqrt{5}y=\frac{4}{3}x+9$ Add $-\sqrt{3}x$ to both sides

$\sqrt{5}y=\frac{4}{3}x-\sqrt{3}x+9=\left(\frac{4}{3}-\sqrt{3}\right)x+9$ Multiply both sides by $\frac{1}{\sqrt{5}}$

$y=\frac{1}{\sqrt{5}}\left(\frac{4}{3}-\sqrt{3}\right)x+\left(\frac{1}{\sqrt{5}}\right)9=\left(\frac{4}{3\sqrt{5}}-\frac{\sqrt{3}}{\sqrt{5}}\right)x+\frac{9}{\sqrt{5}}=\left(\frac{4-3\sqrt{3}}{3\sqrt{5}}\right)x+\frac{9}{\sqrt{5}}$

Red shows what is required. Blue shows some possible further simplification.

33. Find the solution set for $\sqrt{3x+1}=1+\sqrt{x+4}$

$\sqrt{3x+1}=1+\sqrt{x+4}$	Square both sides
$3x+1=1+2\sqrt{x+4}+x+4$	Simplify
$2x-4=2\sqrt{x+4}$	Multiply both sides by $\frac{1}{2}$
$x-2=\sqrt{x+4}$	Square both sides
$x^2-4x+4=x+4$	Add -x-4 to both sides
$x^2-5x=0$	Factor the polynomial
$x(x-5)=0$	Use the Zero Factor Property
$x=0$ OR $x-5=0$	Solve the linear equations
$x=0$ OR $x=5$	Testing 0 and 5 in the original equation is required

TEST 0: $\sqrt{0+1}=1+\sqrt{0+4}$ is FALSE

0 is not a solution of the original equation.

TEST 5: $\sqrt{15+1}=1+\sqrt{5+4}$ is TRUE

5 is a solution of the original equation.

The solution set for the original equation is $\{5\}$.

34. Use the Quadratic Formula to find the solution set for $-x^2 + x - 1 = 0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1^2-(4)(-1)(-1)}}{2(-1)}=\frac{-1\pm\sqrt{1-4}}{-2}=\frac{-1\pm\sqrt{-3}}{-2}=\frac{1\mp\sqrt{-3}}{2}=\frac{1\mp\sqrt{3}i}{2}$$

The solution set is $\left\{\frac{1+\sqrt{3}i}{2},\frac{1-\sqrt{3}i}{2}\right\}$

35. Solve the equation $\boldsymbol{K = \frac{1}{r} - 3m}$ for r.

$\boldsymbol{K = \frac{1}{r} - 3m}$ **Add 3m to both sides**

$K + 3m = \frac{1}{r}$ **Multiply both sides by r**

$r(K + 3m) = 1$ Multiply both sides by $\frac{1}{\text{K + 3m}}$

$r = \frac{1}{\text{K + 3m}}$

36. Compute the difference (–3 + 7i) – (3 + 4i)

(–3 + 7i) – (3 + 4i) = (–3 + 7i) + (–3 – 4i) = (–3 – 3) + (7 + [–4])i = – 6 + 3i

37. Compute the product (2 – 5i)(–7 + 5i)

(2 – 5i)(–7 + 5i) = – 14 +10i + 35i – 25i^2 = - 14 + 45i + 25 = 11 + 45i

38. Compute the quotient (1 + 5i) ÷(3 + 4i)

$$(1+5i)\left(\frac{3-4i}{3^2+4^2}\right) = \frac{(1+5i)(3-4i)}{25} = \frac{3-4i+15i-20i^2}{25} = \frac{23+11i}{25} = \frac{23}{25} + \frac{11}{25}i$$

Red shows what is required. Blue shows some possible further simplification.

39. When solving the equation $5x^3 - 4x^2 + x = 5x^3 + 6$ we might add $-5x^3$ to both sides of the equation to obtain $-4x^2 + x = 6$. The claim is that the two equations are equivalent. What does it mean to say the two equations are equivalent? State a property which justifies the claim.

To say two equations are equivalent means that they have the same solution sets.
If any expression is added to both sides of an equation, the result is an equation which is equivalent to the original equation. In this example the expression $-5x^3$ was added to both sides of the equation $5x^3 - 4x^2 + x = 5x^3 + 6$, so the resulting equation $-4x^2 + x = 6$ is therefore equivalent to the original equation $5x^3 - 4x^2 + x = 5x^3 + 6$.

40. Calculate the area of the shaded triangular portion of the flag show here.
Express the area as sq. in.

$$A = \frac{1}{2}bh = \frac{1}{2}(20)(48) = (10)(48) = 480 \text{ sq.in.}$$