Meramec College Algebra TEST 2 Solution Spring 2006

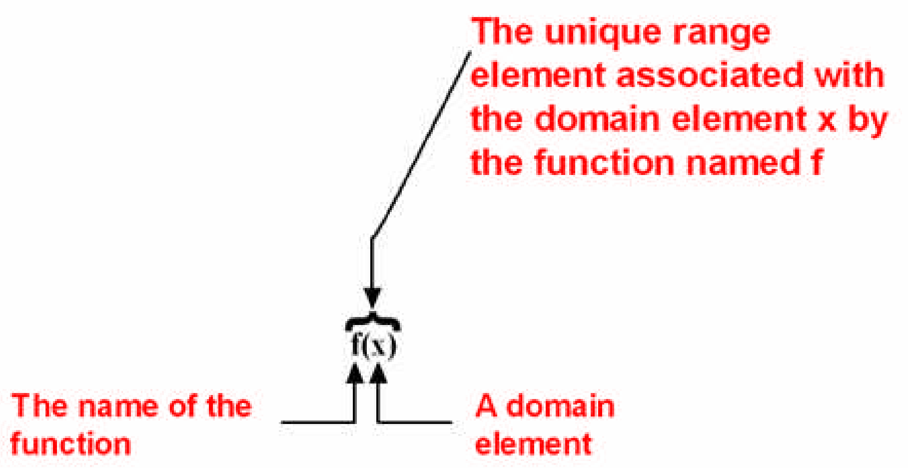
NAME: ______________________________ Score______________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

1. (5 pts) How do you find the zeros of a function? Your response MUST be a complete sentence.
To find the zeros of a function you must solve the equation resulting from f(x) = 0.

2. (5 pts) Fill in the blanks



3. (5 pts) **Definition:** A **function** consists of three things

- A set called the **domain**
- A set called the **range**
- A **rule** which associates **each** element of the domain with a **unique** element of the range.

4. (4 pts) The graph of a function is the set of points whose coordinates satisfy the rule for the function.

Alternate: The graph of a function f is the set of points whose coordinates satisfy (k, f(k)).

5. (4 pts) A **zero of a function** f is a domain element t such that f(t) = 0

.

(10 pts) Rules for functions are given at the top of the page and graphs of functions are given below then. Match the graphs and the rules by writing the letter which identifies a graph in the blank preceding a rule for a function.

6. ___**H**___ $f(x) = 4$

7. ___**E**___ $f(x) = -3x + 3$

8. ___**A**___ $f(x) = x^2$

9. ___**C**___ $f(x) = x^3$

10. ___**F**___ $f(x) = -x^2$

11. ___**J**___ $f(x) = (x + 2)^2$

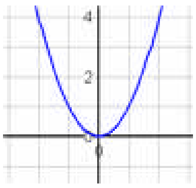
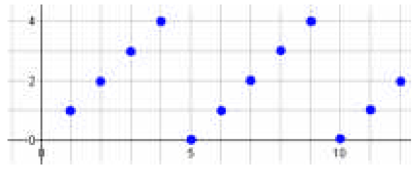
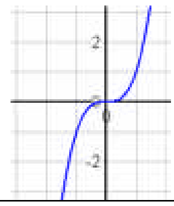
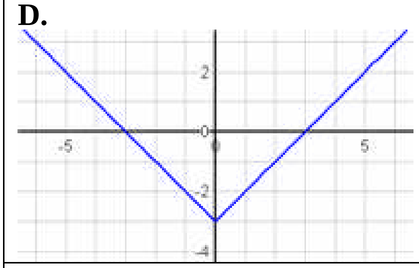
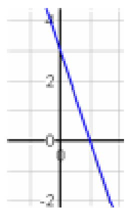
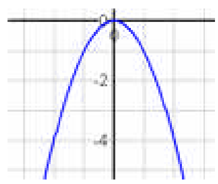
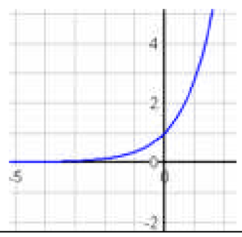
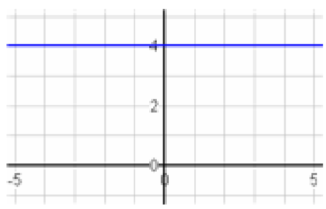
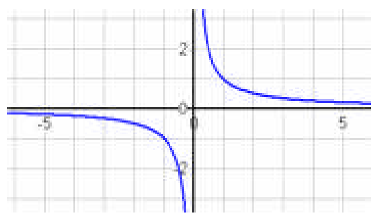
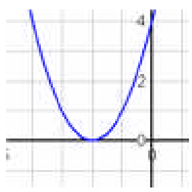
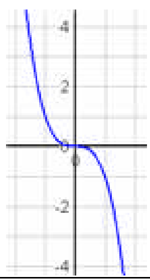
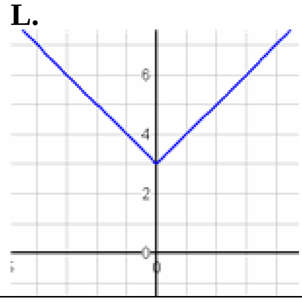
12. ___**D**___ $f(x) = |x| - 3$

13. ___**I**___ $f(x) = \frac{1}{x}$

14. ___**B**___ mod_5

15. ___**G**___ $\exp(x) = e^x$

The graphs are shown in blue.

A. 	**B.** 	**C.** 
D. 	**E.** 	**F.** 
G. 	**H.** 	**I.** 
J. 	**K.** 	**L.** 

16. (4 pts) A linear function is a function whose rule may be written in the form
F(x) = mx + b where m and b are real numbers.

17. (4 pts) A quadratic function is a function whose rule may be written in the form
$f(x) = ax^2 + bx + c$ where a, b, c are real numbers and $a \neq 0$.

18. (4 pts) Suppose f is a function whose rule is $f(x) = (x - 3)(x + 1)(x^2 + x + 1)$. What are the x-intercepts of the graph of f ? The x-intercepts are (3, 0) and (-1, 0).

19. (8 pts) Write the first three terms and the 23rd term of the sequence whose rule is f(n) = 2n – 5. Use functional notation to answer this question.

f(1) = 2(1) – 5 = –3 **f(2) = 2(2) – 5 = –1**
f(3) = 2(3) – 5 = 1 **f(23) = 2(23) – 5 = 41**

20. (8 pts) Find the rule for the linear function f for which f(1) = 3 and f(0) = -4. Show your work and use functional notation.

Because the desired function is linear its rule has the form f(x) = mx + b.

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{3-(-4)}{1-0} = 7$$

So the desired function has the form f(x) = 7x + b.
Now note from the statement of the problem that 3 = f(1) = 7(1) + b.
Solving this equation for b yields b = -4.
Therefore the desired linear function has the rule f(x) = 7x – 4.

21. Find the zeros of the function whose rule is given by $f(x) = x^2 - 2x - 15$.
To find the zeros of a function we must solve the equation resulting from f(x) = 0.
In this case we must solve the equation $0 = f(x) = x^2 - 2x - 15 = (x - 5)(x + 3)$
The solutions to this equation are -3 and 5.
The zero of f are therefore -3 and 5.

22. (8 pts) Solve the equation $x = \sqrt{-5x - 6}$

Square both sides of the equation to obtain
$x^2 = -5x - 6$ Note this equation might not be equivalent to the original equation.
$x^2 + 5x + 6 = 0$
(x + 2)(x + 3) = 0
x = -2 OR x = -3
Neither -2 nor -3 are solutions of the original equation because the principle square root is non-negative.

23. (8 pts) Solve the equation $\frac{x}{x-2} + \frac{1}{x-4} = \frac{2}{(x-4)(x-2)}$

Multiply both sides of the equation by (x – 2)(x – 4) to obtain
x(x – 4) + (x – 2) = 2 **Note this equation might not be equivalent to the original equation.**
$x^2 - 4x + x - 2 = 2$
$x^2 - 3x - 4 = 0$
(x – 4)(x + 1) = 0
Use the Zero Factor Property to decide the equation x(x – 4) + (x – 2) = 2 has solutions 4 and -1.
Because 4 causes one of the denominators of the original equation to be 0, 4 is not a solution to the original equation. However, 0 is a solution of the original equation because it does not cause any denominator to be 0.

24. (8 pts) Consider the arithmetic sequence whose rule is given by f(n) = 2n – 7. Calculate the sum of the first fifty terms. This is usually called the 50^{th} partial sum.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{50} = \frac{50}{2}(a_1 + a_{50}) = \frac{50}{2}(f(1) + f(50)) = 25(-5 + 93) = 25(88)$$

25. (8 pts) The rule for a function is given by $f(x) = \begin{cases} x - 4 & \text{if } -2 < x < 0 \\ x^2 & \text{if } 0 \le x \end{cases}$

What is the domain of f ?
Sketch the graph of f .
What are the zeros of f ?
The domain of f is
$(-2,0) \cup [0,\infty) = (-2,\infty)$
or you could write $\{x \mid -2 < x < \infty\}$

To find the zeros of f we must solve the equation(s) resulting from f(x) = 0.
In this case we must solve the equation x – 4 = 0 to find x = 4.
However according to the rule for f, f(4) = 4^2 = 16.
Therefore 4 is not a zero of the function.
We must also solve the equation x^2 = 0 to find x = 0.
When x = 0, the rule f(x) = x^2 does apply so that f(0) = 0^2 = 0.
Therefore 0 is a zero of the function f.