Course Average_____

Meramec College Algebra TEST 3 Soution Spring 2006

NAME: _____Score_____/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct.

1. **T** F If a vertical line may be drawn so that it intersects a graph in more than one point, then that graph is not the graph of a function.

2. T **F** If a vertical line may be drawn so that it intersects the graph of a function f in more than one point, then the function f does not have an inverse.

3. T **F** If a horizontal line may be drawn so that it intersects a graph in more than one point, then that graph is not the graph of a function.

4. **T** F If a horizontal line may be drawn so that it intersects the graph of a function f in more than one point, then the function f does not have an inverse.

5. **T** F The product of two functions f and g is a function.

6. **T** F Two functions f and g are inverses of each other if $f \circ g(x) = x$ **and** $g \circ f(x) = x$

7. T **F** A function and an equation are the same thing.

8. **T** F Composition of functions is not commutative.

9. **T** F If f and g are functions and f is the inverse of g, then g is the inverse of f.

10. T **F** A quadratic function has an inverse.

11. T **F** The graphs of two third degree polynomials are parallel if they have the same leading coefficient

12. **T** F If 3 is a zero of a polynomial function named f, then $x - 3$ is a factor of the polynomial on the right side of the equality in the rule for f

13. T **F** Every function can be factored into the product of linear and quadratic functions

14. **T** F Every polynomial with real coefficients can be factored into the product of linear and quadratic polynomials with real coefficients.

15. T **F** If $\frac{p}{q}$ is a rational zero of a polynomial function, then p is a divisor of the leading term.

16. **T** F If $\frac{p}{q}$ is a rational zero of a polynomial function, then p is a divisor of the constant term.

17. T **F** The graph of a polynomial function may have sharp corners.

18. T **F** The graph of a fifth degree polynomial function will cross the x-axis exactly five times.

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19. T **F** If $(x-2)^3$ is a factor of a polynomial function f, then the graph of f will touch, but will not cross, the x-axis at 2

20. T **F** A polynomial function and a polynomial equation are the same thing.

21. T **F** If 3i is a zero of a polynomial function f, then (3i, 0) is an x-intercept of the graph of f .

22. T **F** The graph of a polynomial function must cross the x-axis at least once.

23. **T** F The graph of a polynomial function must cross the y-axis at least once.

24. **T** F The graph of a polynomial function of odd degree must cross the x-axis at least once.

25. T **F** The graph of a polynomial function of even degree must cross the x-axis at least once.

26. **T** F A quadratic function is a polynomial function.

27. **T** F The ln and exp functions are inverses of each other.

28. **T** F The log and $\exp_{10}$ functions are inverses of each other.

29. T **F** ln(0) = 1.

30. **T** F exp(1) = e.

31. The sum of two functions f and g is a function f+g whose rule is $(f+g)(x)= f(x) + g(x)$

32. The product of two functions f and g is a function fg whose rule is $(fg)(x)= f(x)g(x)$

33. The composition of two functions f and g is a function $f \circ g$ whose rule is $f \circ g(x) = f\left(g(x)\right)$

34. Two functions f and g are inverses if and only if

$f \circ g(x) = f\left(g(x)\right) = x$ and

$g \circ f(x) = g\left(f(x)\right) = x$

35. A polynomial function is a function whose rule may be written in the form

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ where each a_i is a real number and n is a natural number.

36. For domain elements far from the origin, the **leading** term in a polynomial function dominates the entire expression when calculating **range** elements.

37. The graph of a **polynomial** function is a continuous smooth graph with no sharp corners.

38. If f is a polynomial function such that f(a) < 0and f(b) >0, then f has an **x-intercept** between a and b.

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39. If $\frac{p}{q}$ is a rational zero of a polynomial function, then p is a divisor of the **constant** term.

40. If $\frac{p}{q}$ is a rational zero of a polynomial function, then q is a divisor of the **leading** term.

41. If $3-5i$ is a zero of a polynomial function f, then **3+5i** is a zero of the function f.

If f is a polynomial function whose rule is given by $f(x)=3x^5-2x^4-17x^2+5x+43$, then the following statements in 42-45 are equivalent

42. k is a **real zero** of the function f

43. **k** is a solution of the polynomial equation $3x^5-2x^4-17x^2+5x+43=0$

44. **x - k** is a factor of the polynomial $3x^5-2x^4-17x^2+5x+43$

45. **(k, 0)** is an **x-intercept** of the graph of the function.

46. $\ln\circ\exp(x)=$ **x**

47. $\exp\circ\ln(3x^2-4x+7)=$ **$3x^2-4x+7$**

48. exp is a function whose rule is **$\exp(x)=e^x$**

49. What do we know about the rule for ln? **nothing**

50. Zeros of a quadratic function may be found with the formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

51. Suppose f and g are functions whose rules may be written as f(x) = 3x +1 and $g(x)=x^2-2$. Find the rule for the sum (f + g). Write the rule correctly as a rule.

$(f+g)(x)=f(x)+g(x)=(3x+1)+(x^2-2)=x^2+3x-1$

52. Suppose f and g are functions whose rules may be written as f(x) = 3x +1 and $g(x)=x^2-5$. Find the rule for the composition $f\circ g$. Write the rule correctly as a rule.

$f\circ g(x)=f(g(x))=f(x^2-5)=3(x^2-5)+1=3x^2-14$

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53. Suppose f is the function whose rule is f(x) = 5x – 7. Find its inverse f-1

$y = 5x - 7$

$x = 5y - 7$

$x + 7 = 5y$

$y = \frac{x+7}{5}$

$f^{-1}(x) = \frac{x+7}{5} = \frac{1}{5}x + \frac{7}{5}$

54. Suppose f is a function whose rule is $f(x) = 3x^7 + 8x^6 - 3x^4 + 2x^2 + 11x - 14$. If $\frac{p}{q}$ is a rational zero of f, then $p \in \{\pm 1, \pm 2, \pm 7, \pm 14\}$

$q \in \{\pm 1, \pm 3\}$

and $\frac{p}{q} \in \left\{\pm 1, \pm 2, \pm 7, \pm 14, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{7}{3}, \pm\frac{14}{3}\right\}$

55. Use long division to find the quotient and remainder when

$3x^4 + 2x^3 - 4x^2 - 2x + 7$ is divided by $x^2 + 2x - 1$

$$
\begin{array}{r}
3x^2 - 4x + 7 \\
x^2 + 2x - 1 \overline{)\,3x^4 + 2x^3 - 4x^2 - 2x + 7} \\
\underline{3x^4 + 6x^3 - 3x^2} \\
-4x^3 - x^2 - 2x + 7 \\
\underline{-4x^3 - 8x^2 + 4x} \\
7x^2 - 6x + 7 \\
\underline{7x^2 + 14x - 7} \\
-20x + 14
\end{array}
$$

56. Suppose that f is the function whose rule is $f(x) = x^3 - x^2 + 1$. Use the Intermediate Value Theorem to show that f has an x-intercept between -1 and 0.

f(-1) = –1 – 1 + 1 = -1 < 0

f(0) = 1 > 0

Therefore according to the Intermediate Value Theorem there is a t ∈ (-1, 0) such that f(t) = 0. The point (t, 0) is an x-intercept.

57. Suppose f is a function whose rule is f(x) = (x – 3)(x + 2)(x – 5). Answer the following questions about the function f

a) What kind of function is f ? **f is a third degree polynomial function.**

b) What are the x-intercepts of the graph of f ?
The x-intercepts are (3, 0), (-2, 0), and (5, 0)

c) Describe the behavior of the graph far from the origin .

as $x \to +\infty$, $f(x) \to +\infty$
as $x \to -\infty$, $f(x) \to -\infty$

58. Sketch the graph of the exponential function exp.