

College Algebra TEST 3 Chapter 3 Solution Summer 2005

NAME: _____ Score _____ /100

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SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct. (1 point each)

1. T **F** $3y + 1 = 4x - 7$ is a polynomial function.
2. T **F** The graphs of two third degree polynomials are parallel if they have the same leading coefficient
3. **T** F If 3 is a zero of a polynomial function named f , then $x - 3$ is a factor of the polynomial on the right side of the equality in the rule for f
4. T **F** Every function can be factored into the product of linear and quadratic functions
5. **T** F Every polynomial with real coefficients can be factored into the product of linear and quadratic polynomials.
6. T **F** If $\frac{p}{q}$ is a rational zero of a polynomial function, then p is a divisor of the leading term.
7. **T** F If $\frac{p}{q}$ is a rational zero of a polynomial function, then p is a divisor of the constant term.
8. T **F** The graph of a polynomial function may have sharp corners.
9. T **F** The graph of a fifth degree polynomial function will cross the x -axis exactly five times.
10. T **F** If $(x - 2)^3$ is a factor of a polynomial function f , then the graph of f will touch, but will not cross, the x -axis at 2
11. T **F** A polynomial function and a polynomial equation are the same thing.
12. T **F** If $3i$ is a zero of a polynomial function f , then $(3i, 0)$ is an x -intercept of the graph of f .
13. T **F** The graph of a polynomial function must cross the x -axis at least once.
14. **T** F The graph of a polynomial function must cross the y -axis at least once.
15. **T** F A quadratic function is a polynomial function.

Fill in the blanks (1 point each)

16. A **polynomial function** is a function whose rule may be written in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where each a_i is a real number and n is a natural number.

17. The exponent n of the leading term $a_n x^n$ of the polynomial function f , is called the **degree** of the function f .

18. For domain elements far from the origin, the **leading** term in a polynomial function dominates the entire expression when calculating **range** elements.

19. The graph of a **polynomial** function is a continuous smooth graph with no sharp corners.

20. Let a and b be real numbers such that $a < b$. If f is a polynomial function such that $f(a) \neq f(b)$ then, in the interval $[a, b]$ f takes on every value between **$f(a)$** and **$f(b)$** .

21 & 22. If f is a polynomial function f with integer coefficients, then every rational zero has the form $\frac{p}{q}$ such that:

p is a factor of the **constant** term

q is a factor of the **leading** term

23. If f is a polynomial function such that $f(a) < 0$ and $f(b) > 0$, then f has an **x-intercept** between a and b .

24 – 27. If f is a polynomial function whose rule is given by $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ then the following statements are equivalent.

a is a **zero** of the function f .

a is a solution of the polynomial equation $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = 0$

$x - a$ is a factor of the polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$.

$(a, 0)$ is an x-intercept of the **graph** of the function f .

28. If $3 - 5i$ is a zero of a polynomial function f , then **$3 + 5i$** is a zero of the function f .

29. Zeros of a quadratic formula may be found with the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

30. The vertex of a quadratic function named f is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right)$

SHOW YOUR WORK – 3 points each

31. Find the vertex of the graph of the function whose rule is $f(x) = 3x^2 - 4x + 5$

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right) = \left(\frac{4}{6}, f\left(\frac{4}{6}\right) \right) = \left(\frac{2}{3}, f\left(\frac{2}{3}\right) \right) = \left(\frac{2}{3}, \frac{11}{3} \right)$$

$$\text{Note: } f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^2 - 4\left(\frac{2}{3}\right) + 5 = \frac{4}{3} - \frac{8}{3} + \frac{15}{3} = \frac{11}{3}$$

32. Find zeros of the quadratic function whose rule is $f(x) = x^2 + x + 1$

To find the zeros we must solve the equation resulting from $f(x) = 0$.

So we must solve the quadratic equation $3x^2 - 4x + 5 = 0$ which can be done with the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - (4)(1)(1)}}{(2)(1)} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$$

33. Consider the function whose rule is $f(x) = -2x^2 + x - 5$. There is a point on the graph of f which has 2 as its first coordinate. What is the second coordinate?

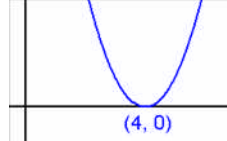
Points on the graph of a function f have the form $(k, f(k))$.

Therefore the desired second coordinate is $f(2) = -2(2^2) + 2 - 5 = -11$

34. Suppose f is a function whose rule is $f(x) = x^2 - 8x + 16$. Sketch the graph of f showing and labeling the x -intercepts and the vertex. Labeling means to give the coordinates.

Note that $f(x) = x^2 - 8x + 16 = (x - 4)^2$. From this we see that f is a quadratic function whose graph is a parabola which opens up and has only one x -intercept at 4.

This x -intercept must therefore be the vertex of the graph.



35. Suppose f is a function whose rule is $f(x) = 6x^7 + 8x^6 - 3x^4 + 2x^2 + 11x - 14$.

If $\frac{p}{q}$ is a rational zero of f , then

$$p \in \{\pm 1, \pm 2, \pm 7, \pm 14\} \qquad \pm q \in \{\pm 1, \pm 2, \pm 3, \pm 6\}$$

$$\text{and } \frac{p}{q} \in \left\{ \pm 1, \pm 2, \pm 7, \pm 14, \pm \frac{1}{2}, \pm \frac{2}{3}, \pm \frac{1}{3}, \pm \frac{7}{2}, \pm \frac{7}{3}, \pm \frac{1}{6}, \pm \frac{7}{6}, \pm \frac{14}{3} \right\}$$

36. Use long division to find the quotient and remainder when

$3x^4 + 2x^3 - 4x^2 - 2x + 7$ is divided by $x^2 + 2x - 1$

$$\begin{array}{r} 3x^2 - 4x + 7 \\ x^2 + 2x - 1 \overline{) 3x^4 + 2x^3 - 4x^2 - 2x + 7} \\ \underline{3x^4 + 6x^3 - 3x^2} \\ -4x^3 - x^2 - 2x + 7 \\ \underline{-4x^3 - 8x^2 + 4x} \\ 7x^2 - 6x + 7 \\ \underline{7x^2 + 14x - 7} \\ -20x + 14 \end{array}$$

37. Suppose that f is the function whose rule is $f(x) = x^3 - x^2 + 1$.

Use the Intermediate Value Theorem to show that f has an x -intercept between -1 and 0 .

Note that

$$f(-1) = (-1)^3 - (-1)^2 + 1 = -1 - 1 + 1 = -1 < 0$$

and $f(0) = 1 > 0$

According to the Intermediate Value Theorem there is an x -intercept in the interval $(-1, 0)$

38. Construct the rule, in factored form, for a polynomial function, which has 3, 7, and -2 as zeros.

$$f(x) = (x - 3)(x - 7)(x + 2)$$

39. Suppose f is the function whose rule is $f(x) = 3x^2 - 2x + k$.

The point $(2, 15)$ is on the graph of f . What is the value of k ?

Because the point $(2, 15)$ is on the graph of f , it follows that $f(2) = 15$.

This yields $15 = f(2) = 3(2^2) - 2(2) + k = 8 + k$. Solve this equation for k to obtain $k = 7$.

40 - 43. Suppose f is a function whose rule is $f(x) = (x - 3)(x + 2)(x - 5)$. Find the zeros of f .

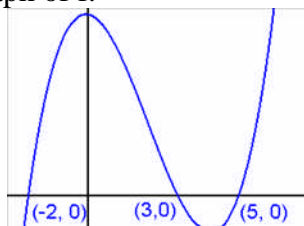
Answer the following questions about the function f . **The zeros are 3, -2, and 5.**

- a) What kind of function is f ? **f is a third degree polynomial function.**
 c) What are the x-intercepts of the graph of f ? **The x-intercepts are (3, 0), (-2, 0), and (5, 0)**
 d) Describe the behavior of the graph far from the origin .

as $x \longrightarrow -\infty$, $f(x) \longrightarrow -\infty$

as $x \longrightarrow +\infty$, $f(x) \longrightarrow +\infty$

f) Sketch the graph of f .



44 - 51 . Suppose f is a function whose rule may be written as $f(x) = x^3 - x^2 + 2$.

Answer the following questions about f .

- a) What kind of function is f ? **It is a third degree polynomial function.**
 b) What can be stated about the graph of f ?

The graph will be a continuous smooth curve with no sharp corners

d) How many x-intercepts do you expect ?

I expect three but know there may be less, but there must be at least one and that there can be no more than three.

e) What are the possible rational zeros of f ?

The possible rational zeros are ± 1 and ± 2

f) What are the zeros of f ? $f(1) = 1 - 1 + 2 \neq 0$ so 1 is not a zero

$f(-1) = -1 - 1 + 2 = 0$, so -1 is a zero and $x + 1$ is a factor. Use ordinary long division to find the quadratic factor to be $x^2 - 2x + 2$. That means $f(x) = x^3 - x^2 + 2 = (x + 1)(x^2 - 2x + 2)$. Use the quadratic formula to find the zeros of the quadratic factor. They are $1 + i$ and $1 - i$.

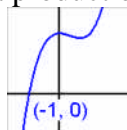
Therefore the zeros of f are -1, $1 + i$, and $1 - i$.

$$\begin{array}{r} x^2 - 2x + 2 \\ x + 1 \overline{) x^3 - x^2 + 2} \\ \underline{x^3 + x^2} \\ -2x^2 + 2 \\ \underline{-2x^2 - 2x} \\ 2x + 2 \\ \underline{2x + 2} \\ 0 \end{array}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

g) What are the x-intercepts of the graph of f ? **(-1, 0)**

h) Write the rule for f as a product of linear and quadratic polynomials. **$f(x) = (x + 1)(x^2 - 2x + 2)$**



j) Sketch the graph of f .