College Algebra TEST 4 Chapter 4 Solution Summer 2005

NAME: ______________________________________Score_____________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct. (2 points each)

1. T **F** The graph of a rational function may intersect its vertical asymptote.

2. **T** F The graph of a rational function may intersect it horizontal asymptote.

3. T **F** Every rational function has at least one vertical asymptote.

4. T **F** Every rational function has at least one real zero.

5. T **F** No rational function has its domain equal to the entire set of real numbers.

6. T **F** Every zero of the numerator of a rational function is a zero of the rational function.

7. **T** F If k is a zero of the denominator of a rational function, then k is not in the domain of the rational function.

8. T **F** The graph of a rational function may have sharp corners.

9. **T** F Some rational functions have no horizontal asymptotes.

10. **T** F The zeros of a rational function are the zeros of its numerator which are not also zeros of the denominator.

11. **T** F If f is a rational function and k is a zero of the numerator but not a zero of the denominator, then k is a zero of f.

12. T **F** If f is a rational function and k is a zero of the denominator but not a zero of the numerator, then k is a zero of f.

13. **T** F If f is a rational function and k is a zero of the denominator but not a zero of the numerator, then x = k is a vertical asymptote of the graph of f.

14. **T** F If f is a rational function and k is a zero of both the numerator and the denominator, then the graph of f will have a hole in it at x = k.

15. **T** F If f is a rational function and k is a zero of the numerator but not a zero of the denominator, then k is a zero of f.

SHOW YOUR WORK – 5 points each

16. What is the domain of the function f whose rule is $f(x)=\dfrac{3x-4}{x+5}$

The domain is all real numbers except the real zeros of the denominator.
The domain of this function is therefore all real numbers except 5.

Set builder notation may be used to describe the domain as $\{x \mid x \in R \text{ and } x \neq 5\}$

Interval notation may be used to describe the domain as $(-\infty,-5)\cup(-5,\infty)$

17. What are the zeros of the function f whose rule is $f(x)=\dfrac{3x-4}{x+5}$

The zeros are the zeros of the numerator which are not zeros of the denominator.

The domain of this function is therefore all real numbers except $\frac{4}{3}$

Set builder notation may be used to describe the domain as $\left\{x \,\middle|\, x \in R \text{ and } x \neq \frac{4}{3}\right\}$

Interval notation may be used to describe the domain as $\left(-\infty,\frac{4}{3}\right)\cup\left(\frac{4}{3},\infty\right)$

18. Suppose f is a rational function which has the following properties:

f has vertical asymptotes at x = -1 and x = 4

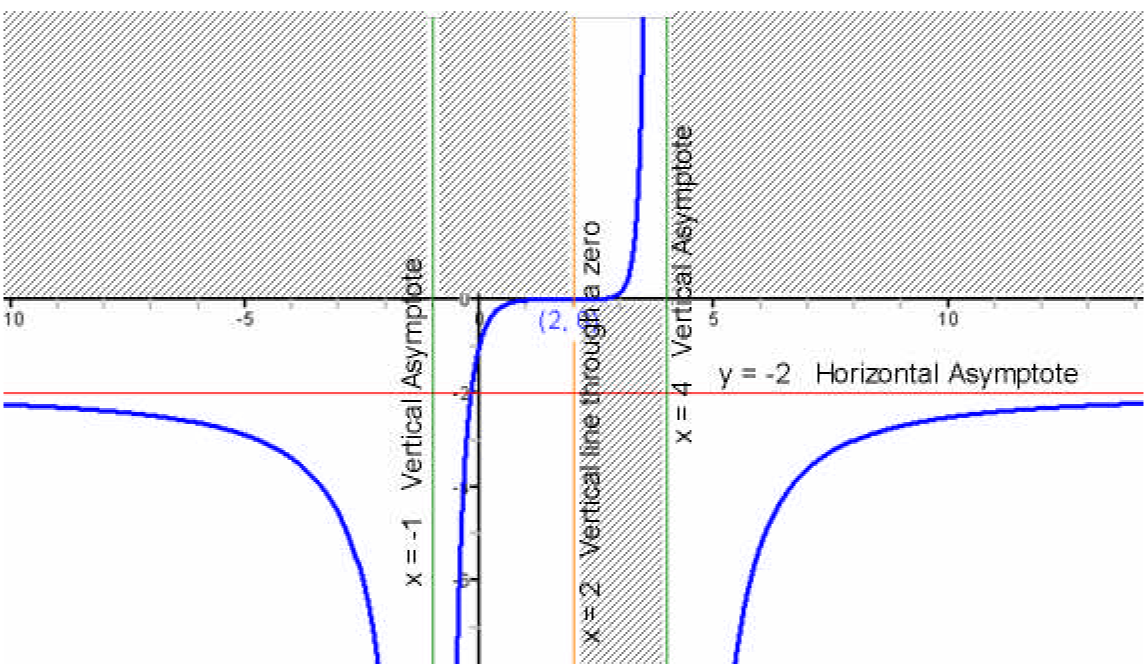
2 is a zero of f   The graph of f intersects its horizontal asymptote one time .

y = -2 is a horizontal asymptote for the graph of f

$f(x) < 0$ if $x \in (-\infty, -1)$ $f(x) < 0$ if $x \in (-1, 2)$

$f(x) > 0$ if $x \in (2, 4)$ $f(x) < 0$ if $x \in (4, +\infty)$

Show the excluded regions and sketch the graph of f. LABEL all important lines and points.



19. Find the horizontal asymptote (if any) of the function whose rule is $f(x)=\dfrac{3x^2-2x+5}{7x^2-16}$

$$f(x)=\frac{3x^2-2x+5}{7x^2-16}=\frac{\frac{3x^2}{x^2}-\frac{2x}{x^2}+\frac{5}{x^2}}{\frac{7x^2}{x^2}-\frac{16}{x^2}}=\frac{3-\frac{2}{x}+\frac{5}{x^2}}{7-\frac{16}{x^2}}\longrightarrow\frac{3}{7}\ \text{ as }\ x\longrightarrow\infty$$

The function f has a horizontal asymptote and it is $y=\dfrac{3}{7}$

20. The line y = 3 is a horizontal asymptote of the rational function whose rule is $f(x)=\dfrac{3x^2+1}{x^2+x+9}$. The graph of f does intersect the horizontal asymptote. Find the point(s) of intersection.

The graph of f intersects the line y = 3 at x such that 3 = f(x).
We must therefore solve the equation

$$\frac{3x^2+1}{x^2+x+9}=3$$

$$3x^2+1=3x^2+3x+27$$

$$1=3x+27$$

$$x=\frac{-26}{3}$$

The point of intersection is therefore $\left(\dfrac{-26}{3},3\right)$

21. The function f whose rule is $f(x)=\dfrac{x+1}{x^2+1}$ has no vertical asymptotes. Explain why not.

Because the denominator has no real zeros.

22. The function f whose rule is $f(x)=\dfrac{x^2+2x+4}{x^2-x-6}$ has no x-intercepts. Explain why not.

The graph of f has x-intercepts at the real zeros of f. The zeros of f are found by solving the equation resulting from f(x) = 0. We therefore must solve the equation

$$x^2+2x+4=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-2\pm\sqrt{4-(4)(1)(4)}}{2}=\frac{-2\pm\sqrt{-12}}{2}$$

Both of these zeros are complex numbers, therefore the graph of f has no x-intercepts.

23. The function f whose rule is $f(x)=\dfrac{x^4-2x^3+x-9}{x^2-3x+2}$ has no horizontal asymptote. Explain why not. **Because the degree of the numerator is greater than the denominator. An alternate method to determine that f has no horizontal asymptote is**

$$f(x)=\frac{x^4-2x^3+x-9}{x^2-3x+2}=\frac{\frac{x^4}{x^2}-\frac{2x^3}{x^2}+\frac{x}{x^2}-\frac{9}{x^2}}{\frac{x^2}{x^2}-\frac{3x}{x^2}+\frac{2}{x^2}}=\frac{x^2-2x+\frac{1}{x}-\frac{9}{x^2}}{1-\frac{3}{x}+\frac{2}{x^2}}\longrightarrow x^2-2x\ \text{ as }\ x\longrightarrow\infty$$

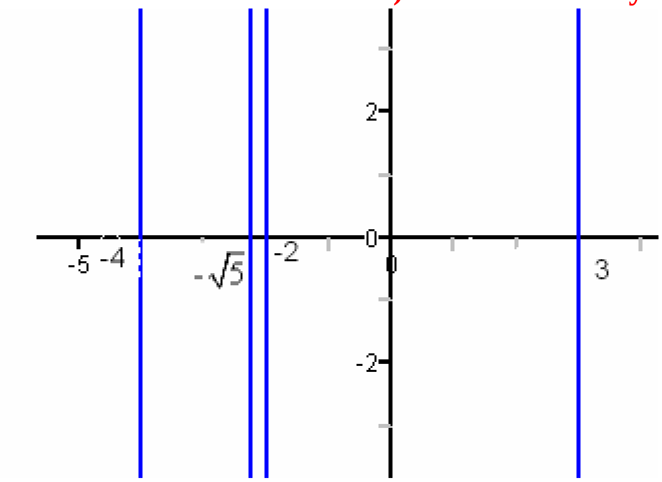
24. Find the xeros of the funtion f whose rule is $f(x) = \dfrac{x^2 - x - 2}{x^3 - 4x^2 - 5x}$

Find the zeros of the numerator:
$0 = x^2 - x - 2 = (x - 2)(x + 1)$ So the zeros of the numerator are 2 and -1
Find the zeros of the denominator:
$0 = x^3 - 4x^2 - 5x = x(x^2 - 4x - 5) = x(x - 5)(x + 1)$ So the zeros of the denominator are 0, 5 and -1.
The zeros of f are the zeros of the numerator which are not zeros of the denominator.
Therefore the zero of f is 2.

25. Find the vertical asymptotes of the function f whose rule is $f(x) = \dfrac{x^2 - x - 2}{x^3 - 4x^2 - 5x}$

The vertical asymptotes occur at the real zeros of the denominator which are not zeros of the numerator.
From the computations in Number 24, we can conclude that the vertical asymptotes of f are x = 0 and x = 5.

26. Find and sketch the strips used to determine the excluded regions for the function f whose rule is $f(x) = \dfrac{(x-3)(x+2)(x+4)}{\left(x+\sqrt{5}\right)\left(x^2+1\right)}$ **The strips are determined by vertical asymptotes and vertical lines though each of the real zeros of the function.**
The real zeros of the function are clearly 3, -2, and -4 (They are zeros of the numerator which are not zeros of the denominator). The vertical asymptote is $x = -\sqrt{5}$



27. Write the rule for a function f which has x-intercepts at 2 and 3 and has x = - 7 as a vertical asymptote.

$f(x) = \dfrac{(x-2)(x-3)}{x+7}$

(10 points)

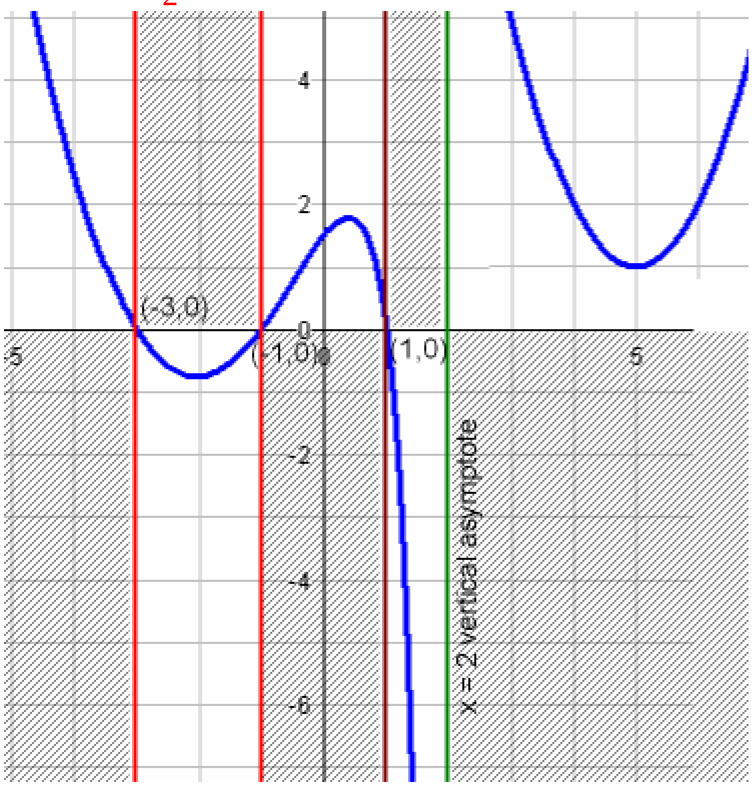
28. Completely analyze the function f whose rule is $f(x)=\dfrac{(x^2-1)(x+3)}{x-2}$. Sketch the graph on the graph paper provided. Put your name on the graph paper.

The zeros of the numerator are 1, -1, and -3. They are all real numbers.
The zero of the denominator is 2. It is a real number
The zeros of f are 1, -1, and -3. x = 2 is a vertical asymptote of the graph.
The graph of f has no horizontal asymptote because the degree of the numerator is greater than the degree of the denominator.

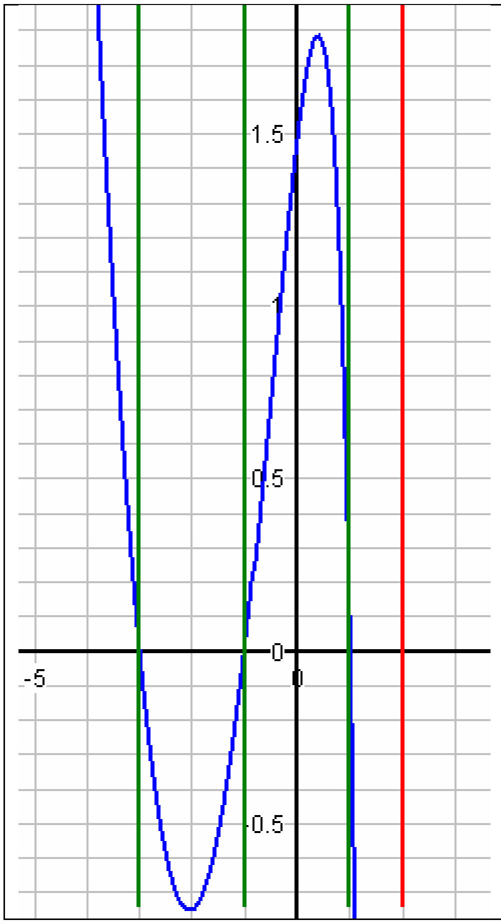
Test (-4): $f(-4)=\dfrac{(16-1)(-1)}{-6}>0$ **Test -2:** $f(-2)=\dfrac{(4-1)(1)}{-4}<0$

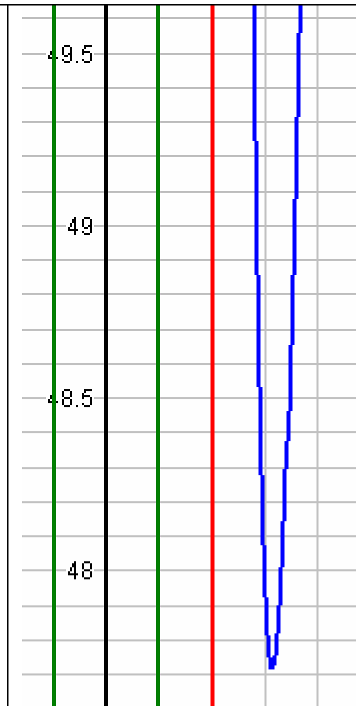
Test 0: $f(0)=\dfrac{(-1)(3)}{-2}>0$ **Test 3:** $f(3)=\dfrac{(8)(6)}{1}>0$

Test $\frac{3}{2}$**:** $f\left(\frac{3}{2}\right)=\dfrac{\left(\frac{9}{4}-1\right)\left(\frac{3}{2}+3\right)}{\frac{3}{2}-2}<0$ **These five tests determine the shading on the graph below.**



The graph on the following page is computer generated. It illustrates that algebraic tools alone are not enough to gain an accurate picture of the function. The calculus provides additional tools for analyzing functions. Those calculus tools are enough to obtain the same graph as the computer generated graph.





Pay close attention to the numbers on the axis to get a true impression of the main part of the graph and its relation to the right-most portion (shown as the insert).