College Algebra **TEST 5 Solution Chapter 5 & 6** **Summer 2005**

1. T **F** The log and exp functions are inverses of each other.
2. **T** F The ln and exp functions are inverses of each other.
3. T **F** $\ln(x + y) = \ln(x) + \ln(y)$.
4. **T** F $k\ln(x) = \ln(x^k)$.
5. T **F** $\ln(0) = 1$.
6. **T** F $\exp(0) = 1$.
7. **T** F $\ln(1) = 0$.
8. T **F** $\exp(1) = 0$.
9. T **F** $e^{x+y} = e^x + e^y$.
10. **T** F $e^{x-y} = \dfrac{e^x}{e^y}$.
11. **T** F If an equation in a system of equations is replaced with an equivalent equation the resulting sytem is equivalent to the original system.
12. **T** F In a system of linear equations, if the value of one of the variables is known, an equivalent system is generated if that value is substituted into the equations.
13. **T** F Two systems of equations are equivalent systems if they have the same solution sets.
14. **T** F The point (0, 0) is in the solution set for the inequality $3x + 2y > -7$
15. T **F** The boundary line is part of the solution set for the inequality $3x + 2y > 5$.
16. If the two functions h and k are inverses of each other, what is the value of $h \circ k(23)$?

    $h \circ k(23) = 23$
17. If f is a function whose rule is $f(x) = 3x - 7$, what is the rule for the function $\exp \circ f$?

    $\exp \circ f(x) = \exp(f(x)) = \exp(3x - 7) = e^{3x-7}$
18. If w is a function whose rule is $w(x) = \dfrac{x-1}{x}$, what is the rule for the function $w \circ \ln$?

    $w \circ \ln(x) = w(\ln(x)) = \dfrac{\ln(x)-1}{\ln(x)}$
19. Write the exponential statement $e^3 = 20.085$ in logarithmic form.

    $3 = \ln(e^3) = \ln(20.085)$
20. Write the logarithmic statement $\ln(4) = 1.386$ in exponential form.

    $4 = \exp(\ln(4)) = \exp(1.386) = e^{1.386}$
21. Use change of base formulas to change $\log_7(13)$ to an expression involving only the ln function.

    $\log_7(13) = \dfrac{\ln(13)}{\ln(7)}$
22. Solve the equation $\ln(2x) = -4$

    $2x = \exp(\ln(x)) = \exp(-4) = e^{-4}$

    $2x = e^{-4}$

    $x = \dfrac{e^{-4}}{2}$
23. Solve the equation $e^{(3x+2)} = 5$

    $3x + 2 = \ln(e^{3x+2}) = \ln(5)$

    $3x = \ln(5) - 2$

    $x = \dfrac{\ln(5) - 2}{3}$

24. Solve the equation $250e^{5x} = 500$

$$e^{5x} = \frac{500}{250} = 2$$
$$5x = \ln\left(e^{5x}\right) = \ln(2)$$
$$x = \frac{\ln(2)}{5}$$

25. Solve the equation $\ln(x) - \ln(x+2) = 4$

$$\ln\left(\frac{x}{x+2}\right) = 4$$
$$\frac{x}{x+2} = \exp\left(\ln\left(\frac{x}{x+2}\right)\right) = \exp(4) = e^4$$
$$x = xe^4 + 2e^4$$
$$x - xe^4 = 2e^4$$
$$x\left(1 - e^4\right) = 2e^4$$
$$x = \frac{2e^4}{1 - e^4}$$

26. Suppose f is a function whose rule is $f(x) = \ln\left(\frac{3x+2}{x^2+5}\right)$

Find a function g so that $f = \ln \circ g$. Write the rule for the function g.

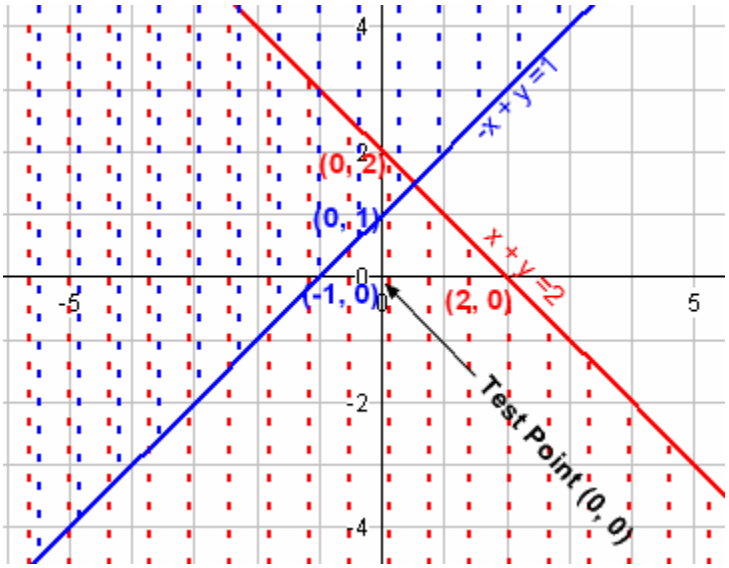
g is the function whose rule is $g(x) = \frac{3x+2}{x^2+5}$

27. Write the equation for the boundary line for the inequality $2x + 3y < 6$. **$2x + 3y = 6$**

28. Sketch the graph of the solution for the system $\begin{cases} x + y \le 2 \\ -x + y \ge 1 \end{cases}$

Show the boundary lines, the important intercepts, and a test point. Show all important computations.

Test (0, 0): $0 + 0 \le 2$ is true $0 + 0 \ge 1$ is false



29. Use substitution (no row operations) to solve the system. $\begin{cases} x-2y=1 \\ 3x-2y=-3 \end{cases}$

$$\begin{cases} x-2y=1 \\ 3x-2y=-3 \end{cases} \longrightarrow \begin{cases} x=2y+1 \\ 3x-2y=-3 \end{cases} \longrightarrow \begin{cases} x=2y+1 \\ 3(2y+1)-2y=-3 \end{cases} \longrightarrow \begin{cases} x=2y+1 \\ 4y+3=-3 \end{cases}$$

$$\longrightarrow \begin{cases} x=2y+1 \\ y=-\frac{3}{2} \end{cases} \longrightarrow \begin{cases} x=-3+1=-2 \\ y=-\frac{3}{2} \end{cases} \longrightarrow \begin{cases} x=2 \\ y=-\frac{3}{2} \end{cases}$$

The solution is the ordered pair $\left(-2,-\frac{3}{2}\right)$

30. Eliminate x in the second equation in this system $\begin{cases} x-2y+z=1 \\ 2x+y-4z=0 \\ x+y-z=8 \end{cases}$

$$\begin{cases} x-2y+z=1 \\ 2x+y-4z=0 \\ x+y-z=8 \end{cases} \xrightarrow{-2R_1+R_2 \longrightarrow R_2} \begin{cases} x-2y+z=1 \\ 5y-6z=-2 \\ x+y-z=8 \end{cases}$$

Alternate:

$$\begin{cases} x-2y+z=1 \\ 2x+y-4z=0 \\ x+y-z=8 \end{cases} \xrightarrow{-2R_3+R_2 \longrightarrow R_2} \begin{cases} x-2y+z=1 \\ -y-2z=-16 \\ x+y-z=8 \end{cases}$$

32. Find the solution to this system $\begin{cases} x-2y+3z=9 \\ y+3z=5 \\ z=2 \end{cases}$

$$\begin{cases} x-2y+3z=9 \\ y+3z=5 \\ z=2 \end{cases} \longrightarrow \begin{cases} x-2y+6=9 \\ y+6=5 \\ z=2 \end{cases} \longrightarrow \begin{cases} x-2y=3 \\ y=-1 \\ z=2 \end{cases} \longrightarrow \begin{cases} x+2=3 \\ y=-1 \\ z=2 \end{cases} \longrightarrow \begin{cases} x=1 \\ y=-1 \\ z=2 \end{cases}$$

The solution is the ordered triple (1, -1, 2)

33. Perform the indicated linear operations

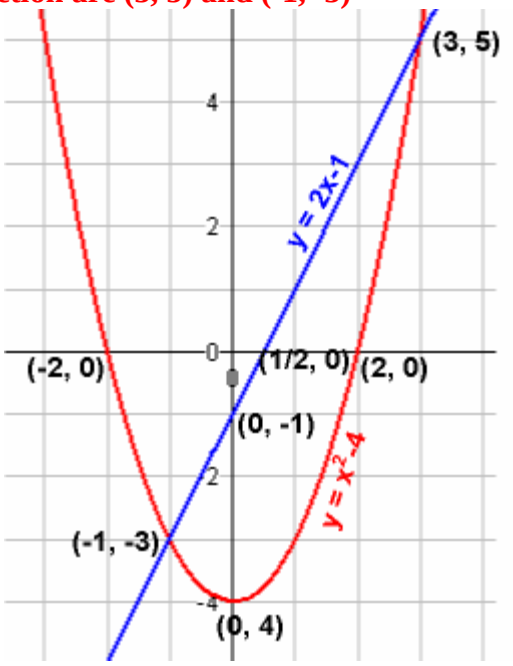
$$\begin{cases} x-y+z=5 \\ 2x-y+2z=1 \\ -x+3y+z=-3 \end{cases} \xrightarrow{-2R_1+R_2 \longrightarrow R_2} \begin{cases} x-y+z=5 \\ y=-9 \\ -x+3y+z=-3 \end{cases}$$

34. On the same coordinate system graph each equation in the following system. Determine the solution to the system. Plot and label the solution on the same coordinate system as the two graphs.

$$\begin{cases} y = x^2 - 4 \\ y = 2x - 1 \end{cases} \longrightarrow \begin{cases} 2x - 1 = x^2 - 4 \\ y = 2x - 1 \end{cases} \longrightarrow \begin{cases} x^2 - 2x - 3 = 0 \\ y = 2x - 1 \end{cases}$$

$$\longrightarrow \begin{cases} (x-3)(x+1) = 0 \\ y = 2x - 1 \end{cases} \longrightarrow \begin{cases} x = 3 \ \textbf{OR} \ x = -1 \\ y = 2x - 1 \end{cases} \longrightarrow \begin{cases} x = 3 \ \textbf{OR} \ x = -1 \\ y = 5 \ \textbf{OR} \ y = -3 \end{cases}$$

The points of intersection are (3, 5) and (-1, -3)



35. Sketch the graph of the exp function.