College Algebra TEST 6 Solution Chapter 7 Summer 2005

1. T **F** Every square matrix has an inverse.
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6. If the two matrices A and B are inverses of each other, then their product is **I**
7. Write the 3×3 identity matrix. $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$
8. Two matrices A and B are equal if they have the same **order** and corresponding **entries** are equal.
9. **Elementary Row Operations:**

   1. Interchange two **rows**
   2. Multiply a row by a **non-zero constant**.
   3. Add a **multiple** of a row to **another** row.

10. Matrix multiplication **is not** commutative.

11. Perform the addition:

$$\begin{bmatrix}2&-2&1\\4&-3&0\end{bmatrix}+\begin{bmatrix}-1&0&5\\-2&-3&1\end{bmatrix}=\begin{bmatrix}1&-2&6\\2&-6&1\end{bmatrix}$$

13. Perform the scalar multipliction:

$$\frac{3}{4}\begin{bmatrix}1&-2&4\\4&-3&2\end{bmatrix}=\begin{bmatrix}\frac{3}{4}&-\frac{3}{2}&3\\3&-\frac{9}{4}&\frac{3}{2}\end{bmatrix}$$

12. Perform the subtraction:

$$\begin{bmatrix}2&-2&1\\4&-3&0\end{bmatrix}-\begin{bmatrix}-1&0&5\\-2&-3&1\end{bmatrix}=\begin{bmatrix}2&-2&1\\4&-3&0\end{bmatrix}+(-1)\begin{bmatrix}-1&0&5\\-2&-3&1\end{bmatrix}$$

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14. Perform the multiplication:

$$\begin{bmatrix}1&2&-3\\4&-2&5\end{bmatrix}\begin{bmatrix}1&-2\\-5&0\\2&7\end{bmatrix}=\begin{bmatrix}-15&-23\\24&27\end{bmatrix}$$

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$$\begin{bmatrix}1&2&-1\\-3&1&-2\\-1&3&1\end{bmatrix}\begin{bmatrix}2&0&3\\3&2&4\\-1&5&-1\end{bmatrix}=\begin{bmatrix}9&-1&12\\-1&-8&-3\\6&11&8\end{bmatrix}$$

16. Verify that A and B are inverses of each other: $A=\begin{bmatrix}3&-1\\-2&2\end{bmatrix}$ and $B=\begin{bmatrix}\frac{1}{2}&\frac{1}{4}\\\frac{1}{2}&\frac{3}{4}\end{bmatrix}$

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17. Find the inverse of A :Use only elementary row operations. $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 7 & 0 & 1 \end{array}\right] \xrightarrow{-3R_1 + R_2 \longrightarrow R_2} \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & -3 & 1 \end{array}\right] \xrightarrow{-2R_2 + R_1 \longrightarrow R_1} \left[\begin{array}{cc|cc} 1 & 0 & 7 & -2 \\ 0 & 1 & -3 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

18. Consider the matrices. $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $C = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$

Solve the matrix equation AX = C.

$$X = A^{-1}AX = A^{-1}C$$

Thus $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = X = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -20 \\ 20 \\ -9 \end{bmatrix}$ **From which it follows that x= -20, y = 20 and z = -9**

19. With elementary row operations, use the 1 in the 22 position to get a zero in the 12 position **THEN STOP**

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20. What does the following matrix equality mean ?

$$\begin{bmatrix} x + 2y & z - 2y \\ x + w + z & 8 \end{bmatrix} = \begin{bmatrix} 2 & w - 3 \\ 4 & x - z \end{bmatrix}$$

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$$\begin{bmatrix} x+2y & z-2y \\ x+w+z & 8 \end{bmatrix} = \begin{bmatrix} 2 & w-3 \\ 4 & x-z \end{bmatrix}$$

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College Algebra **TEST 6 Solution Chapter 7** Summer 2005

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$$\begin{bmatrix} 2 & -2 & 1 \\ 4 & -3 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 5 \\ -2 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 6 \\ 2 & -6 & 1 \end{bmatrix}$$

13. Perform the scalar multipliction:

$$\frac{3}{4}\begin{bmatrix} 1 & -2 & 4 \\ 4 & -3 & 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & -\frac{3}{2} & 3 \\ 3 & -\frac{9}{4} & \frac{3}{2} \end{bmatrix}$$

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$$\begin{bmatrix} 2 & -2 & 1 \\ 4 & -3 & 0 \end{bmatrix} - \begin{bmatrix} -1 & 0 & 5 \\ -2 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \\ 4 & -3 & 0 \end{bmatrix} + (-1)\begin{bmatrix} -1 & 0 & 5 \\ -2 & -3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -2 & 1 \\ 4 & -3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -5 \\ 2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 6 & 0 & -1 \end{bmatrix}$$

14. Perform the multiplication:

$$\begin{bmatrix} 1 & 2 & -3 \\ 4 & -2 & 5 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -5 & 0 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} -15 & -23 \\ 24 & 27 \end{bmatrix}$$

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16. Verify that A and B are inverses of each other: $A = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$

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18. Consider the matrices. $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $C = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$

Solve the matrix equation AX = C.

$$X = A^{-1}AX = A^{-1}C$$

Thus $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = X = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -20 \\ 20 \\ -9 \end{bmatrix}$ **From which it follows that x= -20, y = 20 and z = -9**

19. With elementary row operations, use the 1 in the 22 position to get a zero in the 12 position **THEN STOP**

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