Meramec College Algebra **TEST 2 Solution** **Summer 2006**

NAME: ______________________________ Score______________________/100

Please print

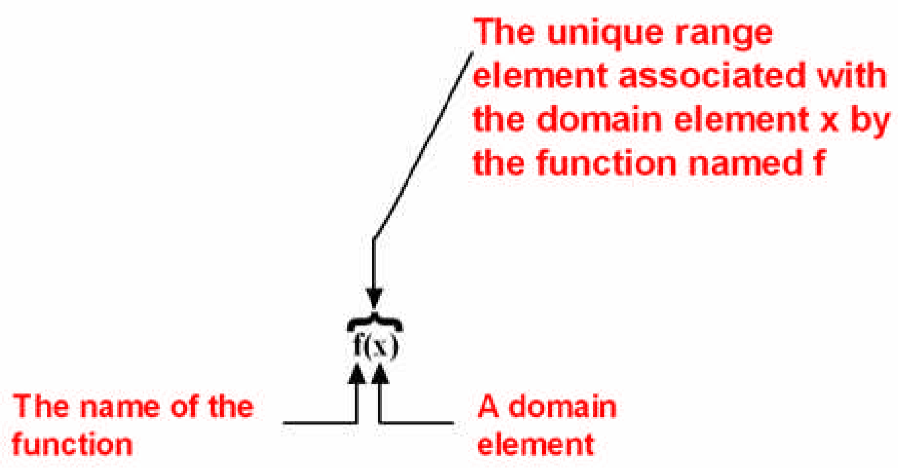
SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Use = symbols when appropriate. Do not use = symbols when not appropriate.

Questions 1 – 10 are each worth 2 points.

1. **T** F The zeros of a function are domain elements.
2. T **F** The zeros of a function are range elements.
3. T **F** The function whose rule is $f(x) = |3x - 5|$ is a linear function .
4. **T** F The graph of a function f is above the x-axis if $f(x) > 0$ for all domain elements x.
5. T **F** The rule for every function may be written as an equation.
6. **T** F If f is a function and k is a domain element then the point (k, f(k)) is on the graph of f.
7. **T** F The product of two functions is a function.
8. **T** F The squaring function is a quadratic function.
9. T **F** Every function has an inverse.
10. **T** F The composition of two functions is a function

11. **(5 pts)**



12. **(5 pts) Definition: A function consists of three things**

- **A set called the domain**
- **A set called the range**
- **A rule which associates each element of the domain with a unique element of the range.**

(10 pts) Rules for functions are given at the top of the page and graphs of functions are given below them. Match the graphs and the rules by writing the letter which identifies a graph in the blank preceding a rule for a function.

13. ___H___ $f(x) = 4$

14. ___E___ $f(x) = -3x + 3$

15. ___C___ $f(x) = x^2$

16. ___K___ $f(x) = x^3$

17. ___F___ $f(x) = -x^2$

18. ___I___ $f(x) = (x + 2)^2$

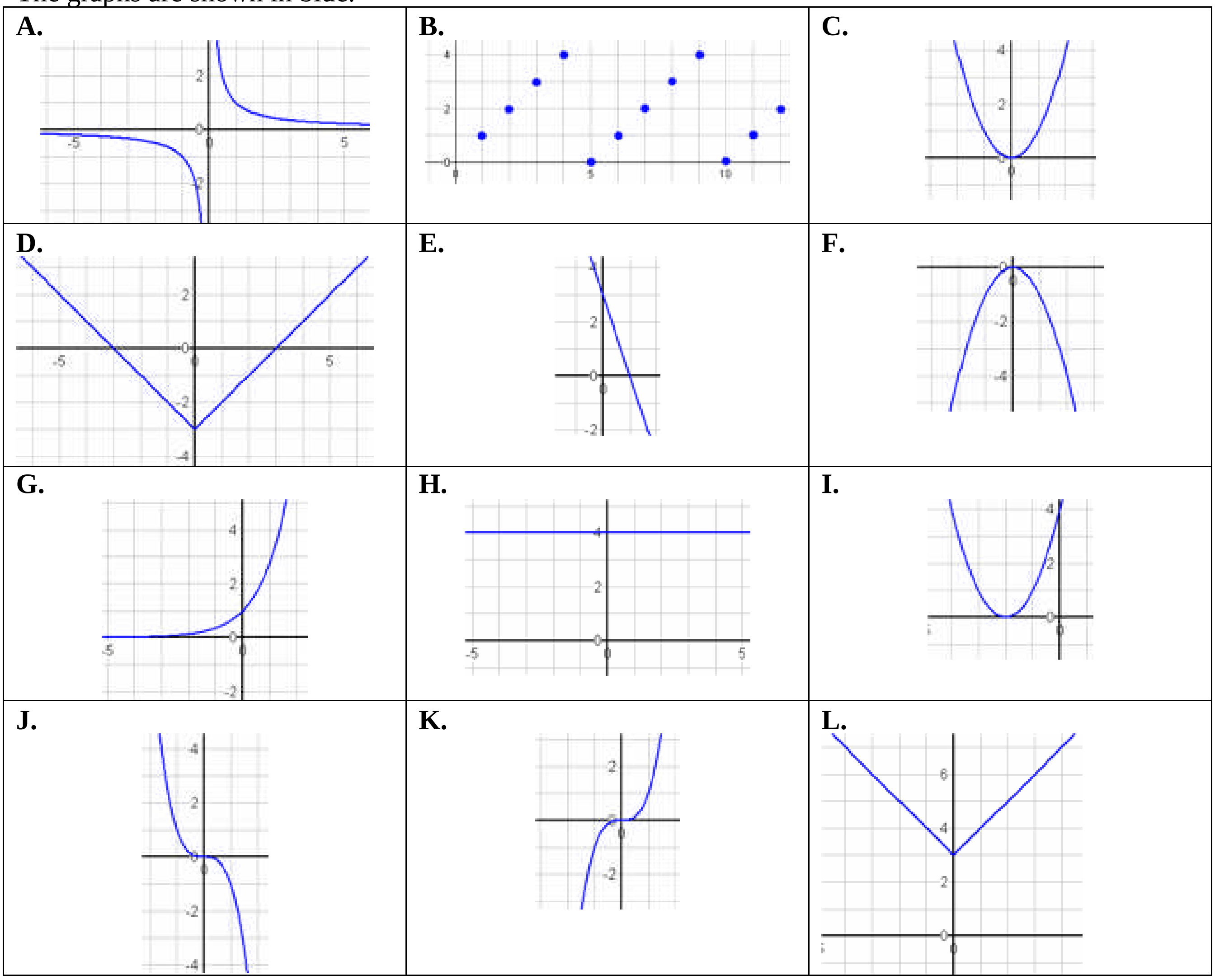
19. ___D___ $f(x) = |x| - 3$

20. ___A___ $f(x) = \dfrac{1}{x}$

21. ___B___ mod_5

22. ___G___ $\exp(x) = e^x$

The graphs are shown in blue.



Questions 23 – 39 are each worth 3 points.

23. A linear function is a function whose rule may be written in the form

f(x) = mx + b where m and b are real numbers.

24. A quadratic function is a function whose rule may be written in the form

f(x) = ax^2 + bx + c where a, b, and c are real numbers and a ≠ 0.

25. Suppose f is a function whose rule is f(x) = x^2 + 3x + 1 and k – m is in the domain of f. Compute the unique range element associated with the domain element k – m.

$f(k-m) = (k-m)^2 + 3(k-m) + 1 = k^2 - 2km + m^2 + 3k - 3m + 1$

26. Find the rule for the linear function f for which f(1) = 3 and f(0) = – 4.
Hint: Determine the slope and the y-intercept and then write the rule.
Because the desired function is linear it has the form f(x) = mx + b where m is the slope and b is the y-intercept.
Because f(0) = – 4 the y-intercept is – 4.
Two points on the graph of f are (1, 3) and (0, – 4) from which we can compute the slope

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-4-3}{0-1} = 7$$

The rule for the desired function is f(x) = 7x – 4

27. Suppose the rule for the function f is f(x) = x^3 – 2x + 4. Compute $f\left(\frac{1}{2}\right)$ NO DECIMALS

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right) + 4 = \frac{1}{8} - 1 + 4 = \frac{25}{8}$$

28. Find the zeros of the function whose rule is $f(x) = \frac{2}{3}x - \frac{4}{9}$. NO DECIMALS

The zeros of f are found by solving the equation resulting from f(x) = 0.

$$\frac{2}{3}x - \frac{4}{9} = 0$$

So we solve $\frac{2}{3}x = \frac{4}{9}$

$$x = \left(\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{9}}}\right)\left(\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}\right) = \frac{2}{3}$$

The zero of f is $\frac{2}{3}$

29. The rules for two functions f and g are given by f(x) = 2x – 5 and g(x) = x^2 – 2x – 15.
Find the rules for each of the following functions. Make sure you use functional notation and your rule is written as a rule

a) fg

$$fg(x) = (f(x))(g(x)) = (2x-5)(x^2-2x-15) = 2x^3 - 4x^2 - 30x - 5x^2 + 10x + 75$$
$$= 2x^3 - 9x^2 - 20x + 75$$

b) $f \circ g$

$$f \circ g(x) = f(g(x)) = f(x^2 - 2x - 15) = 2(x^2 - 2x - 15) - 5 = 2x^2 - 4x - 35$$

c) $g \circ f$

$g \circ f(x) = g(f(x)) = g(2x-5) = (2x-5)^2 - 2(2x-5) - 15 = 4x^2 - 24x + 20$

d) f^{-1} Use the standard process to find the inverse

$f(x) = 2x - 5$

$y = 2x - 5$

$x = 2y - 5$

$2y = x + 5$

$y = \frac{x+5}{2}$

So $f^{-1}(x) = \frac{x+5}{2}$

30. Fill in the blanks to find the zeros of the function whose rule is given by $f(x) = x^2 - 2x - 15$.

Solution: To find the **zeros** of f we must solve the **equation** resulting from **f(x) = 0**

Therefore in this case we must solve $0 = x^2 - 2x - 15 = (x-5)(x+3)$

The **Zero Factor Property** assures us the solution set for the equation is $\{-3, 5\}$

Therefore the **zeros** of f are **– 3 and 5**

31. Determine the domain of the function whose rule is $f(x) = \sqrt{-5x-6}$

Hint: Your work should lead to the conclusion that the domain of f is $\left(-\infty, -\frac{6}{5}\right]$

The domain of f is all real numbers for which the rule makes sense. In this case then the domain is all real numbers for which $-5x - 6 \geq 0$ which is equivalent to $-5x \geq 6$ which is equivalent to $x \leq -\frac{6}{5}$.

Therefore the domain is the ray $\left(-\infty, -\frac{6}{5}\right]$

32. The domain of the function whose rule is $f(x) = \frac{x}{x-2} + \frac{1}{x-4}$ is all real numbers except 2 and 4.

Use interval notation to complete the following description of the domain of f.

The domain of f is $(-\infty, 2) \cup (2, 4) \cup (4, \infty)$

33. What is the rule for the linear function whose graph passes through (2, 7) with slope $\frac{3}{5}$

Method 1	Method 2
Use the point slope form of the equation of a line $y - y_1 = m(x - x_1)$ $y - 7 = \frac{3}{5}(x-2)$ $y = \frac{3}{5}(x-2) + 7 = \frac{3}{5}x - \frac{6}{5} + \frac{35}{5} = \frac{3}{5}x + \frac{29}{5}$ The rule for the desired function is $f(x) = \frac{3}{5}x + \frac{29}{5}$	The desired function has the form f(x) = mx + b where m is the slope and b is the y-intercept. So the desired function has the form $f(x) = \frac{3}{5}x + b$ Since the point (2, 7) is on the graph we have $7 = \frac{3}{5}(2) + b$ from which it follows that b = $\frac{29}{5}$ And the desired function is $f(x) = \frac{3}{5}x + \frac{29}{5}$

34 G is the sequence whose rule is given by: G(n) is the number of distinct primes used in the unique prime factorization of n. Compute G(60).

The prime factorization of 60 is $60 = 2^2 \bullet 3 \bullet 5$ which contains three distinct primes.
Therefore G(60) = 3.

35 G is the sequence whose rule is given by: G(n) is the number of distinct primes used in the unique prime factorization of n. What is the domain of G ? **Because G is a sequence its domain is N, the set of natural numbers.**

36. The composition of two functions f and g is a function $f \circ g$ whose rule is $f \circ g(x) = f\left(g(x)\right)$

37. Recall that $mod_6(n)$ is the remainder when n is divided by 6.
Recall that $\tau(n)$ the number of positive natural number divisors of n.

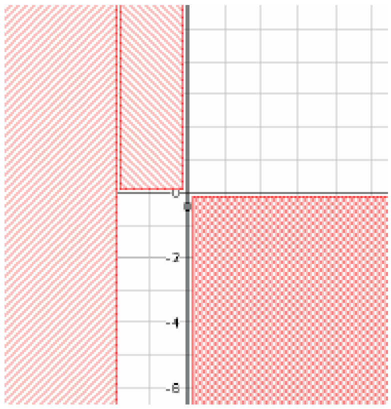
Compute $\tau \circ \text{mod}_6(17)$. Show your work.

17 = 6(2) + 5 implies that $mod_6(17) = 5$. The only natural number divisors of 5 are 1 and 5, so $\tau(5) = 2$

Then $\tau \circ \text{mod}_6(17) = \tau\left(\text{mod}_6(17)\right) = \tau(5) = 2$

38. Suppose f is a function for which:
i) The domain of f is (-2, ∞)
ii) f(x) > 0 for all x ∈ [0, ∞).
iii) f(x) < 0 for all x ∈ (-2, 0)

On the following coordinate system cross off (Shade) the regions which CANNOT contain any part of the graph of f.



39. Sketch the graph of the function whose rule is $f(x) = \left|(x-1)(x+2)\right|$