

Please print

**SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION****Use = symbols when appropriate. Do not use = symbols when not appropriate.****Questions 1 – 25 are each worth 1 point.**

1. T F The graph of a rational function may intersect its vertical asymptotes.
2. T F The graph of a rational function may intersect its horizontal asymptote.
3. T F Every rational function has at least one vertical asymptote.
4. T F Every rational function has at least one real zero.
5. T F No rational function has its domain equal to the entire set of real numbers.
6. T F If  $k$  is a zero of the denominator of a rational function, then  $k$  is not in the domain of the rational function.
7. T F Some rational functions have no horizontal asymptotes.
8. T F If  $f$  is a rational function and  $k$  is a zero of the numerator but not a zero of the denominator, then  $k$  is a zero of  $f$ .
9. T F If  $f$  is a rational function and  $k$  is a zero of the denominator but not a zero of the numerator, then  $k$  is a zero of  $f$ .
10. T F If  $f$  is a rational function and  $k$  is a zero of the denominator but not a zero of the numerator, then  $x = k$  is a vertical asymptote of the graph of  $f$ .
11. T F If  $f$  is a rational function and  $k$  is a zero of the denominator but not a zero of the numerator, then  $y = k$  must be the horizontal asymptote of the graph of  $f$ .
12. T F If  $3$  is a zero of a polynomial function named  $f$ , then  $x - 3$  is a factor of the polynomial on the right side of the equality in the rule for  $f$ .
13. T F If  $\frac{p}{q}$  is a rational zero of a polynomial function, then  $p$  is a divisor of the leading term.
14. T F If  $\frac{p}{q}$  is a rational zero of a polynomial function, then  $p$  is a divisor of the constant term.
15. T F The graph of a fifth degree polynomial function will cross the  $x$ -axis exactly five times.
16. T F If  $(x - 2)^3$  is a factor of a polynomial function  $f$ , then the graph of  $f$  will touch, but will not cross, the  $x$ -axis at  $2$ .
17. T F If  $3i$  is a zero of a polynomial function  $f$ , then  $(3i, 0)$  is an  $x$ -intercept of the graph of  $f$ .
18. T F The graph of a polynomial function must cross the  $x$ -axis at least once.

19. T F The graph of a polynomial function must cross the y-axis at least once.
20. T F The ln and exp functions are inverses of each other.
21. T F  $\ln(x + y) = \ln(x) + \ln(y)$ .
22. T F  $\ln(0) = 1$ .
23. T F  $\exp(0) = 1$ .
24. T F  $\ln(1) = 0$ .
25. T F  $\exp(1) = 0$ .

**Questions 26 – 46 are each worth 3 points UNLESS otherwise noted.**

26. The rule for the exponential function base e is \_\_\_\_\_
27. The rule for the exponential function base 17 is \_\_\_\_\_
28. The exponent n of the leading term  $a_n x^n$  of the polynomial function f, is called the \_\_\_\_\_ of the function f.
29. If f is a polynomial function such that  $f(a) < 0$  and  $f(b) > 0$ , then f has an \_\_\_\_\_ between a and b.
30. If f is a polynomial function f with integer coefficients, then every rational zero has the form  $\frac{p}{q}$  such that:
- a) p is a factor of the \_\_\_\_\_ term
- b) q is a factor of the \_\_\_\_\_ term
31. If the two functions h and k are inverses of each other, what is the value of  $h \circ k(23)$ ?
32. If f is a function whose rule is  $f(x) = 3x - 7$ , what is the rule for the function  $\exp \circ f$  ?  
 $\exp \circ f(x) =$
33. If w is a function whose rule is  $w(x) = \frac{x-1}{x}$ , what is the rule for the function  $w \circ \ln$  ?  
 $w \circ \ln(x) =$
34. Write the exponential statement  $e^3 = 20.085$  in logarithmic form.

35. Write the logarithmic statement  $\ln(4) = 1.386$  in exponential form.

36. Solve the equation  $\ln(2x) = -4$

37. Solve the equation  $e^{(3x+2)} = 5$

38. Solve the equation  $\ln(x) - \ln(x+2) = 4$

39. What is the horizontal asymptote of the function whose rule is  $f(x) = \frac{3x^5 + 2x^3 - 4}{2x^5 + 6}$

40. The function  $f$  whose rule is  $f(x) = \frac{x^2 + 5}{x - 2}$  has no  $x$ -intercepts. Explain why not.

41. **(8 points)** If  $f$  is a polynomial function whose rule is given by

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$  then the following statements are equivalent.

a)  $k$  is a \_\_\_\_\_ of the function  $f$ .

b) \_\_\_\_\_ is a solution of the polynomial equation \_\_\_\_\_.

c) \_\_\_\_\_ is a factor of the polynomial \_\_\_\_\_.

d) \_\_\_\_\_ is an  $x$ -intercept of the \_\_\_\_\_ of the function  $f$ .

42. **(6 points)** Suppose  $f$  is a function whose rule is  $f(x) = 3x^7 + 8x^6 - 3x^4 + 2x^2 + 11x - 5$ .

If  $\frac{p}{q}$  is a rational zero of  $f$ , then

$p \in \{ \quad \quad \quad \} \quad q \in \{ \quad \quad \quad \}$

and  $\frac{p}{q} \in \{ \quad \quad \quad \}$

43. The function  $f$  whose rule is  $f(x) = \frac{3x^7 - 2x^3 + 5x + 7}{x^4 + 16}$  has no horizontal asymptote.

Explain why not

44. Use long division to find the quotient and remainder when

$3x^4 + 2x^3 - 4x^2 - 2x + 7$  is divided by  $x^2 + 2x - 1$

45. (5 points) Suppose  $f$  is a rational function which has the following properties:

$f$  has vertical asymptotes at  $x = -1$  and  $x = 4$

$2$  is a zero of  $f$

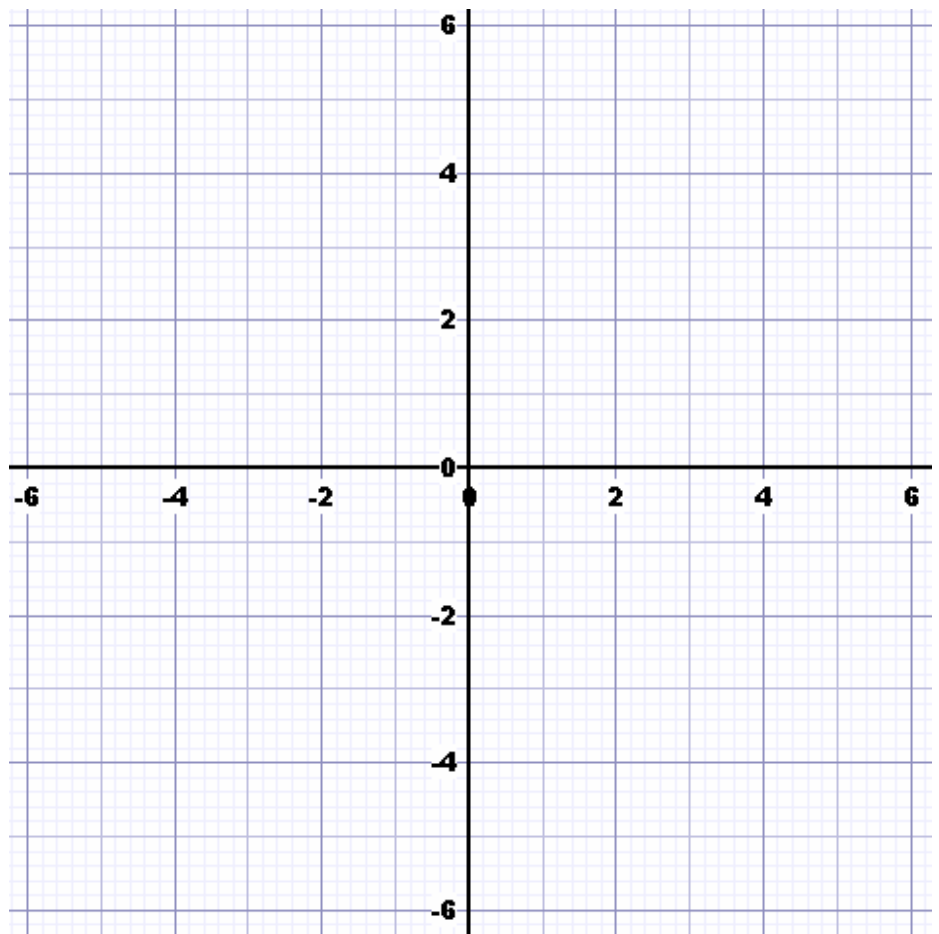
$y = -2$  is a horizontal asymptote for the graph of  $f$

The graph of  $f$  intersects its horizontal asymptote at  $(0, -2)$

$f(x) < 0$  if  $x \in (-\infty, -1)$        $f(x) < 0$  if  $x \in (-1, 2)$

$f(x) > 0$  if  $x \in (2, 4)$        $f(x) < 0$  if  $x \in (4, +\infty)$

Show the excluded regions and sketch the graph of  $f$ . LABEL all important lines and points  
All the properties listed above should be reflected in you drawing on the coordinate system.

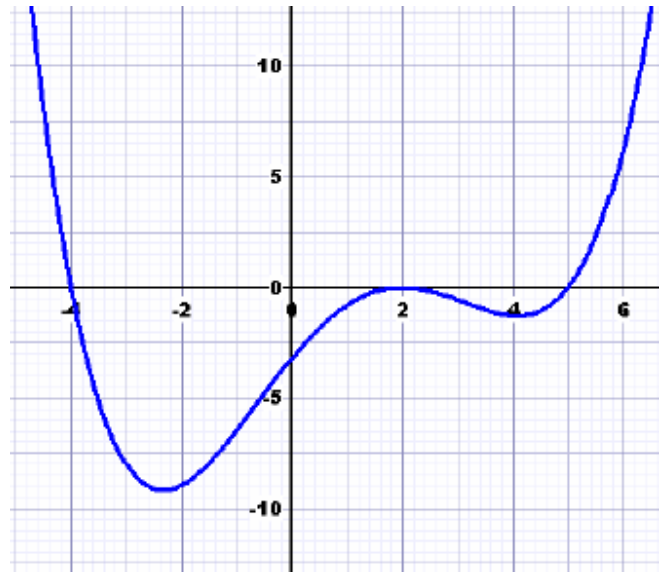


46. (5 points) The graph of a function  $f$  is shown at the right. Answer the following questions about  $f$ .

a) What are the real zeros of  $f$ ?

b) Where is  $f(x) > 0$ ? Use interval notation.

c) Where is  $f(x) < 0$ ? Use interval notation.



d) Discuss the multiplicities of each of the real zeros.

e) As  $x \rightarrow \infty$ ,  $f(x) \rightarrow$  \_\_\_\_\_  
As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow$  \_\_\_\_\_

f) This appears to be the graph of a \_\_\_\_\_ function.

g) If you needed to guess at a rule for the function what would be your best guess?