

Please print

**SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION****Use = symbols when appropriate. Do not use = symbols when not appropriate.****Questions 1 – 25 are each worth 1 point.**

1. T **F** The graph of a rational function may intersect its vertical asymptotes.
2. **T** F The graph of a rational function may intersect its horizontal asymptote.
3. T **F** Every rational function has at least one vertical asymptote.
4. T **F** Every rational function has at least one real zero.
5. T **F** No rational function has its domain equal to the entire set of real numbers.
6. **T** F If  $k$  is a zero of the denominator of a rational function, then  $k$  is not in the domain of the rational function.
7. **T** F Some rational functions have no horizontal asymptotes.
8. **T** F If  $f$  is a rational function and  $k$  is a zero of the numerator but not a zero of the denominator, then  $k$  is a zero of  $f$ .
9. T **F** If  $f$  is a rational function and  $k$  is a zero of the denominator but not a zero of the numerator, then  $k$  is a zero of  $f$ .
10. **T** F If  $f$  is a rational function and  $k$  is a zero of the denominator but not a zero of the numerator, then  $x = k$  is a vertical asymptote of the graph of  $f$ .
11. T **F** If  $f$  is a rational function and  $k$  is a zero of the denominator but not a zero of the numerator, then  $y = k$  must be the horizontal asymptote of the graph of  $f$ .
12. **T** F If  $3$  is a zero of a polynomial function named  $f$ , then  $x - 3$  is a factor of the polynomial on the right side of the equality in the rule for  $f$
13. T **F** If  $\frac{p}{q}$  is a rational zero of a polynomial function, then  $p$  is a divisor of the leading term.
14. **T** F If  $\frac{p}{q}$  is a rational zero of a polynomial function, then  $p$  is a divisor of the constant term.
15. T **F** The graph of a fifth degree polynomial function will cross the  $x$ -axis exactly five times.
16. T **F** If  $(x - 2)^3$  is a factor of a polynomial function  $f$ , then the graph of  $f$  will touch, but will not cross, the  $x$ -axis at  $2$
17. T **F** If  $3i$  is a zero of a polynomial function  $f$ , then  $(3i, 0)$  is an  $x$ -intercept of the graph of  $f$ .
18. T **F** The graph of a polynomial function must cross the  $x$ -axis at least once.

19. **T** F The graph of a polynomial function must cross the y-axis at least once.
20. **T** F The ln and exp functions are inverses of each other.
21. T **F**  $\ln(x + y) = \ln(x) + \ln(y)$ .
22. T **F**  $\ln(0) = 1$ .
23. **T** F  $\exp(0) = 1$ .
24. **T** F  $\ln(1) = 0$ .
25. T **F**  $\exp(1) = 0$ .

**Questions 26 – 46 are each worth 3 points UNLESS otherwise noted.**

26. The rule for the exponential function base e is  **$\exp(x) = e^x$**
27. The rule for the exponential function base 17 is  **$\exp_{17}(x) = 17^x$**
28. The exponent n of the leading term  $a_n x^n$  of the polynomial function f, is called the **degree** of the function f.
29. If f is a polynomial function such that  $f(a) < 0$  and  $f(b) > 0$ , then f has an **x-intercept or real zero or zero** between a and b.
30. If f is a polynomial function f with integer coefficients, then every rational zero has the form  $\frac{p}{q}$  such that:
- p is a factor of the **constant** term
  - q is a factor of the **leading** term
31. If the two functions h and k are inverses of each other, what is the value of  **$h \circ k(23)$** ?  
 **$h \circ k(23) = 23$**
32. If f is a function whose rule is  $f(x) = 3x - 7$ , what is the rule for the function  **$\exp \circ f$**  ?  
 **$\exp \circ f(x) = \exp(f(x)) = \exp(3x - 7) = e^{3x-7}$**
33. If w is a function whose rule is  $w(x) = \frac{x-1}{x}$ , what is the rule for the function  **$w \circ \ln$**  ?  
 **$w \circ \ln(x) = w(\ln(x)) = \frac{\ln(x)-1}{\ln(x)}$**
34. Write the exponential statement  $e^3 = 20.085$  in logarithmic form.  
 **$\exp(3) = 20.085$**   
 **$\ln(\exp(3)) = \ln(20.085)$**   
 **$\ln \circ \exp(3) = \ln(20.085)$**   
 **$3 = \ln(20.085)$**

35. Write the logarithmic statement  $\ln(4) = 1.386$  in exponential form.

$$\exp(\ln(4)) = \exp(1.386)$$

$$\exp \circ \ln(4) = e^{1.386}$$

$$4 = e^{1.386}$$

36. Solve the equation  $\ln(2x) = -4$

$$\exp(\ln(2x)) = \exp(-4)$$

$$\exp \circ \ln(2x) = e^{-4}$$

$$2x = \frac{1}{e^4}$$

$$x = \left(\frac{1}{2}\right)\left(\frac{1}{e^4}\right) = \frac{1}{2e^4}$$

37. Solve the equation  $e^{(3x+2)} = 5$

$$\exp(3x+2) = 5$$

$$\ln(\exp(3x+2)) = \ln(5)$$

$$\ln \circ \exp(3x+2) = \ln(5)$$

$$3x+2 = \ln(5)$$

$$x = \frac{\ln(5) - 2}{3}$$

38. Solve the equation  $\ln(x) - \ln(x+2) = 4$

$$\ln\left(\frac{x}{x+2}\right) = 4$$

$$\exp\left(\ln\left(\frac{x}{x+2}\right)\right) = \exp(4)$$

$$\exp \circ \ln\left(\frac{x}{x+2}\right) = e^4$$

$$\frac{x}{x+2} = e^4$$

$$x = (x+2)e^4 \quad \text{This equation may not be equivalent to the previous equation}$$

$$x = e^4 x + 2e^4$$

$$x - e^4 x = 2e^4$$

$$x(1 - e^4) = 2e^4$$

$$x = \frac{2e^4}{1 - e^4} \quad \text{This does not make the denominator of the fraction } \frac{x}{x+2} \text{ equal 0, so it is a solution.}$$

39. What is the horizontal asymptote of the function whose rule is  $f(x) = \frac{3x^5 + 2x^3 - 4}{2x^5 + 6}$

**The horizontal asymptote is  $y = \frac{3}{2}$  because the numerator and denominator have the same degree.**

40. The function  $f$  whose rule is  $f(x) = \frac{x^2 + 5}{x - 2}$  has no  $x$ -intercepts. Explain why not.

**The numerator is the sum of a square and a positive number, and therefore cannot be 0.**

**or**

**$x^2 \geq 0$ , so  $x^2 + 5 > 0$  for all real numbers  $x$**

**or**

**$x$ -intercepts of the graph of  $f$  would correspond to real zeros of the function  $f$  which would correspond to real zeros of the numerator. The numerator has no real zeros.**

**or**

$$x^2 + 5 = 0$$

$$x^2 = -5$$

$$x = \pm\sqrt{5}i$$

**The only zeros are complex numbers which are not on the  $x$ -axis**

41. (8 points) If  $f$  is a polynomial function whose rule is given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$
 then the following statements are equivalent.

a)  $k$  is a **real zero** of the function  $f$ .

b)  $k$  is a solution of the polynomial equation  $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = 0$

c)  $x - k$  is a factor of the polynomial  $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ .

d) **( $k, 0$ )** is an  $x$ -intercept of the **graph** of the function  $f$ .

42. (6 points) Suppose  $f$  is a function whose rule is  $f(x) = 3x^7 + 8x^6 - 3x^4 + 2x^2 + 11x - 5$ .

If  $\frac{p}{q}$  is a rational zero of  $f$ , then

$$p \in \{\pm 1, \pm 5\} \quad q \in \{\pm 1, \pm 3\}$$

$$\text{and } \frac{p}{q} \in \left\{ \pm 1, \pm 5, \pm \frac{1}{3}, \pm \frac{5}{3} \right\}$$

43. The function  $f$  whose rule is  $f(x) = \frac{3x^7 - 2x^3 + 5x + 7}{x^4 + 16}$  has no horizontal asymptote.

Explain why not

**The degree of the numerator is greater than the degree of the denominator.**

44. Use long division to find the quotient and remainder when  $3x^4 + 2x^3 - 4x^2 - 2x + 7$  is divided by  $x^2 + 2x - 1$

$$\begin{array}{r}
 3x^2 - 4x + 7 \\
 x^2 + 2x - 1 \overline{) 3x^4 + 2x^3 - 4x^2 - 2x + 7} \\
 \underline{3x^4 + 6x^3 - 3x^2} \phantom{- 2x + 7} \\
 -4x^3 - x^2 - 2x + 7 \\
 \underline{-4x^3 - 8x^2 + 4x} \phantom{+ 7} \\
 7x^2 - 6x + 7 \\
 \underline{7x^2 + 14x - 7} \\
 -20x + 14
 \end{array}$$

45. (5 points) Suppose  $f$  is a rational function which has the following properties:

$f$  has vertical asymptotes at  $x = -1$  and  $x = 4$

2 is a zero of  $f$

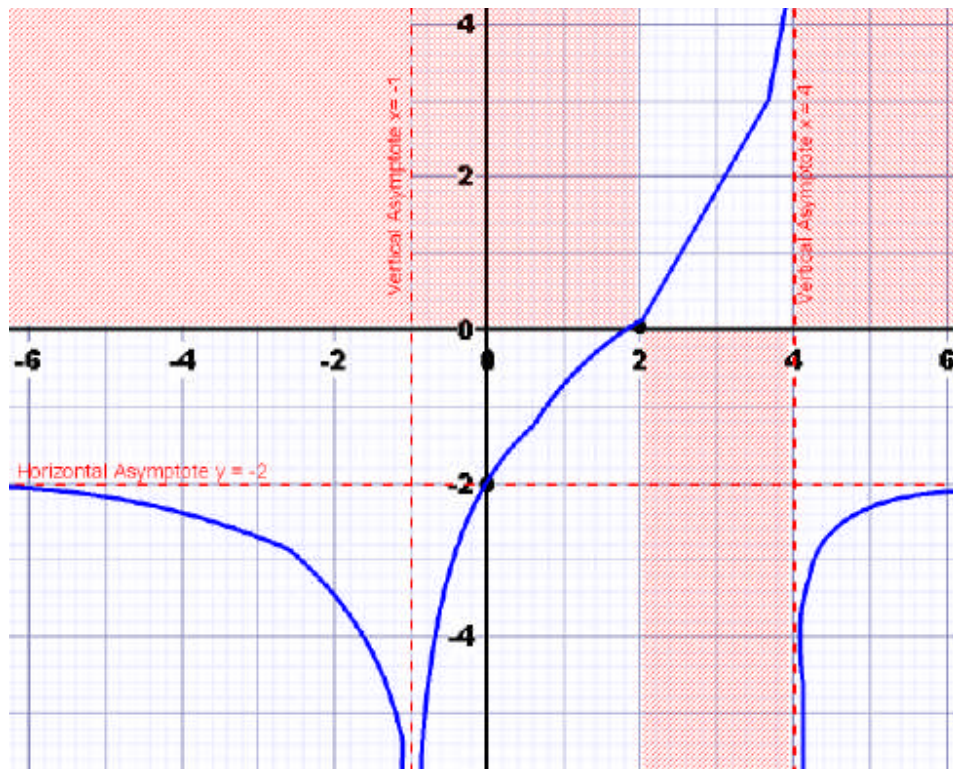
$y = -2$  is a horizontal asymptote for the graph of  $f$

The graph of  $f$  intersects its horizontal asymptote at  $(0, -2)$

$f(x) < 0$  if  $x \in (-\infty, -1)$        $f(x) < 0$  if  $x \in (-1, 2)$

$f(x) > 0$  if  $x \in (2, 4)$        $f(x) < 0$  if  $x \in (4, +\infty)$

Show the excluded regions and sketch the graph of  $f$ . LABEL all important lines and points  
All the properties listed above should be reflected in you drawing on the coordinate system.



46. (5 points) The graph of a function  $f$  is shown at the right. Answer the following questions about  $f$ .

a) What are the real zeros of  $f$ ?

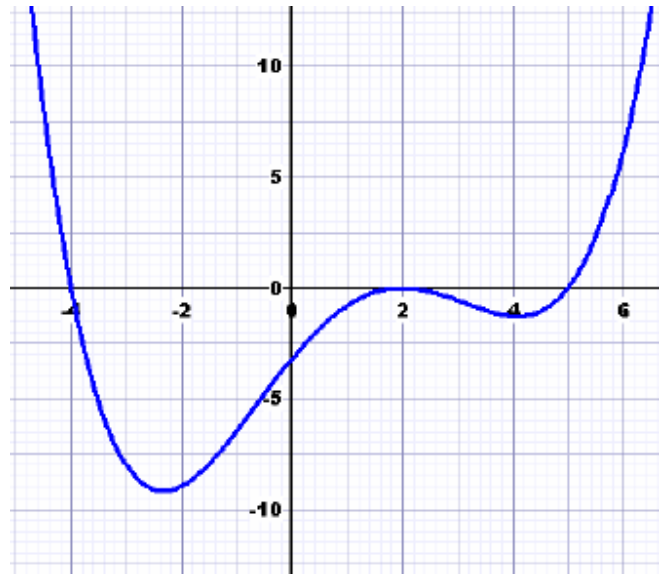
**The real zeros are -4, 2 and 5**

b) Where is  $f(x) > 0$ ? Use interval notation.

**$f(x) > 0$  for  $x \in (-\infty, -4) \cup (5, \infty)$**

c) Where is  $f(x) < 0$ ? Use interval notation.

**$f(x) < 0$  for  $x \in (-4, 2) \cup (2, 5)$**



d) Discuss the multiplicities of each of the real zeros.

**The multiplicities of -4 and 5 are both odd numbers.**

**The multiplicity of 2 is an even number.**

e) As  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$

As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow \infty$

f) This appears to be the graph of a **polynomial** function.

**If this is the graph of a polynomial its degree is no less than 4 and its degree must be an even number.**

g) If you needed to guess at a rule for the function what would be your best guess?

**$f(x) = (x + 4)(x - 2)^2(x - 5)$**