

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION**Use = symbols when appropriate. Do not use = symbols when not appropriate.****Questions 1 – 25 are each worth 1 point.**

1. T **F** The graph of a rational function may intersect its vertical asymptotes.
2. **T** F The graph of a rational function may intersect its horizontal asymptote.
3. T **F** Every rational function has at least one vertical asymptote.
4. T **F** Every rational function has at least one real zero.
5. T **F** No rational function has its domain equal to the entire set of real numbers.
6. **T** F If k is a zero of the denominator of a rational function, then k is not in the domain of the rational function.
7. **T** F Some rational functions have no horizontal asymptotes.
8. **T** F If f is a rational function and k is a zero of the numerator but not a zero of the denominator, then k is a zero of f .
9. T **F** If f is a rational function and k is a zero of the denominator but not a zero of the numerator, then k is a zero of f .
10. **T** F If f is a rational function and k is a zero of the denominator but not a zero of the numerator, then $x = k$ is a vertical asymptote of the graph of f .
11. T **F** If f is a rational function and k is a zero of the denominator but not a zero of the numerator, then $y = k$ must be the horizontal asymptote of the graph of f .
12. **T** F If 3 is a zero of a polynomial function named f , then $x - 3$ is a factor of the polynomial on the right side of the equality in the rule for f
13. T **F** If $\frac{p}{q}$ is a rational zero of a polynomial function, then p is a divisor of the leading term.
14. **T** F If $\frac{p}{q}$ is a rational zero of a polynomial function, then p is a divisor of the constant term.
15. T **F** The graph of a fifth degree polynomial function will cross the x -axis exactly five times.
16. T **F** If $(x - 2)^3$ is a factor of a polynomial function f , then the graph of f will touch, but will not cross, the x -axis at 2
17. T **F** If $3i$ is a zero of a polynomial function f , then $(3i, 0)$ is an x -intercept of the graph of f .
18. T **F** The graph of a polynomial function must cross the x -axis at least once.

19. **T** F The graph of a polynomial function must cross the y-axis at least once.
20. **T** F The ln and exp functions are inverses of each other.
21. T **F** $\ln(x + y) = \ln(x) + \ln(y)$.
22. T **F** $\ln(0) = 1$.
23. **T** F $\exp(0) = 1$.
24. **T** F $\ln(1) = 0$.
25. T **F** $\exp(1) = 0$.

Questions 26 – 46 are each worth 3 points UNLESS otherwise noted.

26. The rule for the exponential function base e is **$\exp(x) = e^x$**
27. The rule for the exponential function base 17 is **$\exp_{17}(x) = 17^x$**
28. The exponent n of the leading term $a_n x^n$ of the polynomial function f, is called the **degree** of the function f.
29. If f is a polynomial function such that $f(a) < 0$ and $f(b) > 0$, then f has an **x-intercept or real zero or zero** between a and b.
30. If f is a polynomial function f with integer coefficients, then every rational zero has the form $\frac{p}{q}$ such that:
- p is a factor of the **constant** term
 - q is a factor of the **leading** term
31. If the two functions h and k are inverses of each other, what is the value of **$h \circ k(23)$** ?
 $h \circ k(23) = 23$
32. If f is a function whose rule is $f(x) = 3x - 7$, what is the rule for the function **$\exp \circ f$** ?
 $\exp \circ f(x) = \exp(f(x)) = \exp(3x - 7) = e^{3x-7}$
33. If w is a function whose rule is $w(x) = \frac{x-1}{x}$, what is the rule for the function **$w \circ \ln$** ?
 $w \circ \ln(x) = w(\ln(x)) = \frac{\ln(x)-1}{\ln(x)}$
34. Write the exponential statement $e^3 = 20.085$ in logarithmic form.
 $\exp(3) = 20.085$
 $\ln(\exp(3)) = \ln(20.085)$
 $\ln \circ \exp(3) = \ln(20.085)$
 $3 = \ln(20.085)$

35. Write the logarithmic statement $\ln(4) = 1.386$ in exponential form.

$$\exp(\ln(4)) = \exp(1.386)$$

$$\exp \circ \ln(4) = e^{1.386}$$

$$4 = e^{1.386}$$

36. Solve the equation $\ln(2x) = -4$

$$\exp(\ln(2x)) = \exp(-4)$$

$$\exp \circ \ln(2x) = e^{-4}$$

$$2x = \frac{1}{e^4}$$

$$x = \left(\frac{1}{2}\right)\left(\frac{1}{e^4}\right) = \frac{1}{2e^4}$$

37. Solve the equation $e^{(3x+2)} = 5$

$$\exp(3x+2) = 5$$

$$\ln(\exp(3x+2)) = \ln(5)$$

$$\ln \circ \exp(3x+2) = \ln(5)$$

$$3x+2 = \ln(5)$$

$$x = \frac{\ln(5) - 2}{3}$$

38. Solve the equation $\ln(x) - \ln(x+2) = 4$

$$\ln\left(\frac{x}{x+2}\right) = 4$$

$$\exp\left(\ln\left(\frac{x}{x+2}\right)\right) = \exp(4)$$

$$\exp \circ \ln\left(\frac{x}{x+2}\right) = e^4$$

$$\frac{x}{x+2} = e^4$$

$$x = (x+2)e^4 \quad \text{This equation may not be equivalent to the previous equation}$$

$$x = e^4 x + 2e^4$$

$$x - e^4 x = 2e^4$$

$$x(1 - e^4) = 2e^4$$

$$x = \frac{2e^4}{1 - e^4} \quad \text{This does not make the denominator of the fraction } \frac{x}{x+2} \text{ equal 0, so it is a solution.}$$

39. What is the horizontal asymptote of the function whose rule is $f(x) = \frac{3x^5 + 2x^3 - 4}{2x^5 + 6}$

The horizontal asymptote is $y = \frac{3}{2}$ because the numerator and denominator have the same degree.

40. The function f whose rule is $f(x) = \frac{x^2 + 5}{x - 2}$ has no x -intercepts. Explain why not.

The numerator is the sum of a square and a positive number, and therefore cannot be 0.

or

$x^2 \geq 0$, so $x^2 + 5 > 0$ for all real numbers x

or

x -intercepts of the graph of f would correspond to real zeros of the function f which would correspond to real zeros of the numerator. The numerator has no real zeros.

or

$$x^2 + 5 = 0$$

$$x^2 = -5$$

$$x = \pm\sqrt{5}i$$

The only zeros are complex numbers which are not on the x -axis

41. (8 points) If f is a polynomial function whose rule is given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$
 then the following statements are equivalent.

a) k is a **real zero** of the function f .

b) k is a solution of the polynomial equation $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 = 0$

c) $x - k$ is a factor of the polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$.

d) **($k, 0$)** is an x -intercept of the **graph** of the function f .

42. (6 points) Suppose f is a function whose rule is $f(x) = 3x^7 + 8x^6 - 3x^4 + 2x^2 + 11x - 5$.

If $\frac{p}{q}$ is a rational zero of f , then

$$p \in \{\pm 1, \pm 5\} \quad q \in \{\pm 1, \pm 3\}$$

$$\text{and } \frac{p}{q} \in \left\{ \pm 1, \pm 5, \pm \frac{1}{3}, \pm \frac{5}{3} \right\}$$

43. The function f whose rule is $f(x) = \frac{3x^7 - 2x^3 + 5x + 7}{x^4 + 16}$ has no horizontal asymptote.

Explain why not

The degree of the numerator is greater than the degree of the denominator.

44. Use long division to find the quotient and remainder when $3x^4 + 2x^3 - 4x^2 - 2x + 7$ is divided by $x^2 + 2x - 1$

$$\begin{array}{r}
 3x^2 - 4x + 7 \\
 x^2 + 2x - 1 \overline{) 3x^4 + 2x^3 - 4x^2 - 2x + 7} \\
 \underline{3x^4 + 6x^3 - 3x^2} \\
 -4x^3 - x^2 - 2x + 7 \\
 \underline{-4x^3 - 8x^2 + 4x} \\
 7x^2 - 6x + 7 \\
 \underline{7x^2 + 14x - 7} \\
 -20x + 14
 \end{array}$$

45. (5 points) Suppose f is a rational function which has the following properties:

f has vertical asymptotes at $x = -1$ and $x = 4$

2 is a zero of f

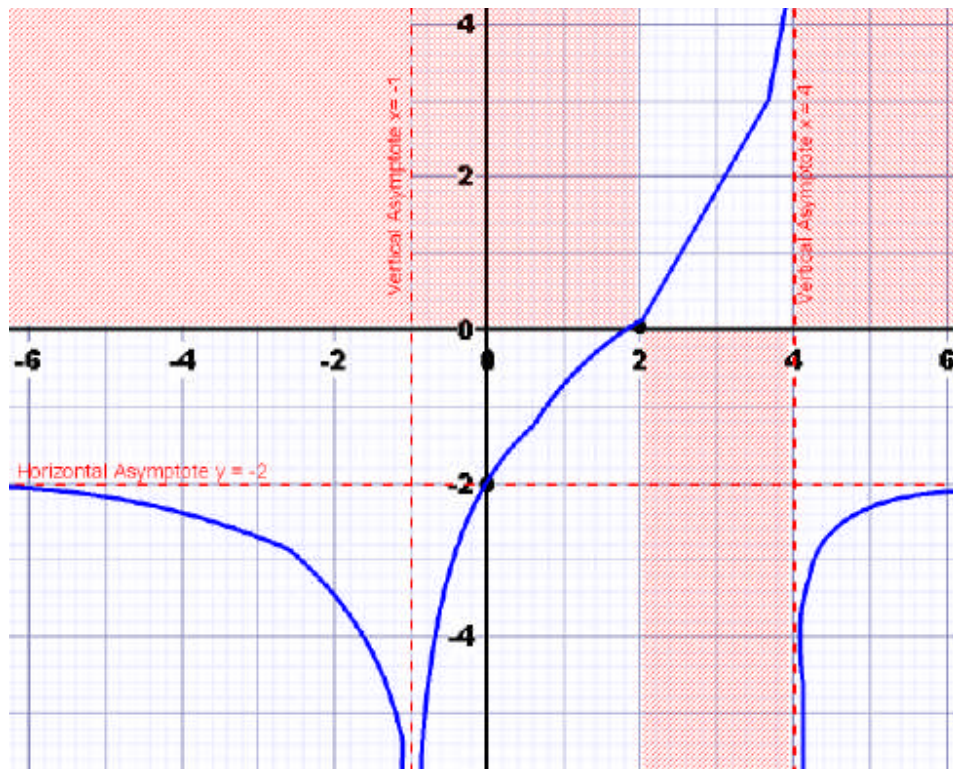
$y = -2$ is a horizontal asymptote for the graph of f

The graph of f intersects its horizontal asymptote at $(0, -2)$

$f(x) < 0$ if $x \in (-\infty, -1)$ $f(x) < 0$ if $x \in (-1, 2)$

$f(x) > 0$ if $x \in (2, 4)$ $f(x) < 0$ if $x \in (4, +\infty)$

Show the excluded regions and sketch the graph of f . LABEL all important lines and points
All the properties listed above should be reflected in your drawing on the coordinate system.



46. **(5 points)** The graph of a function f is shown at the right. Answer the following questions about f .

a) What are the real zeros of f ?

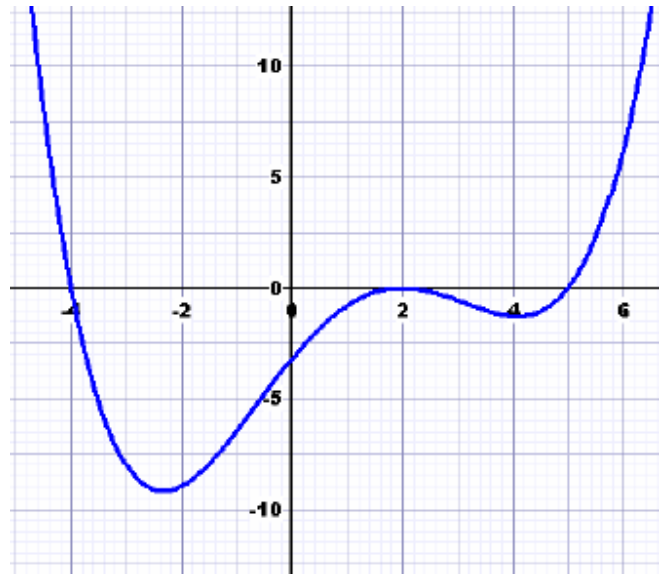
The real zeros are -4, 2 and 5

b) Where is $f(x) > 0$? Use interval notation.

$f(x) > 0$ for $x \in (-\infty, -4) \cup (5, \infty)$

c) Where is $f(x) < 0$? Use interval notation.

$f(x) < 0$ for $x \in (-4, 2) \cup (2, 5)$



d) Discuss the multiplicities of each of the real zeros.

The multiplicities of -4 and 5 are both odd numbers.

The multiplicity of 2 is an even number.

e) As $x \longrightarrow \infty, f(x) \longrightarrow \infty$

As $x \longrightarrow -\infty, f(x) \longrightarrow \infty$

f) This appears to be the graph of a **polynomial** function.

If this is the graph of a polynomial its degree is no less than 4 and its degree must be an even number.

g) If you needed to guess at a rule for the function what would be your best guess?

$f(x) = (x + 4)(x - 2)^2(x - 5)$