Get Good at Mathematics

Recent e-mails and other communications have raised three issues related to studying and learning mathematics.

1) I am not very good at math.
2) How should I study for your tests.
3) How do you want me to write the answer.

1) I am not very good at math

It is a myth that some people are inherently unable to learn mathematics. The myth is fostered by an age old misunderstanding caused by a failure of the education system and a societal willingness to accept poor performance in mathematics. Many studies, including a seven year study by Dr. M. Poage and myself, demonstrate beyond a shadow of a doubt that any student can learn mathematics to a 95% mastery level. Stop telling yourself that your cannot learn mathematics. Stop excusing poor performance by claiming a lack of ability in mathematics. Address your poor performance honestly and initiate corrective action.

Poor performance in mathematics is almost always the direct result of insufficiently studying the requisite material. Correct this error and you will start doing well in mathematics. The key to addressing this issue is to recognize what causes your study efforts to be insufficient. The two most common causes are: (a) ignorance of prerequisite material, (b) insufficient study time, and (c) inappropriate study methods.

(b) Prerequisites

Mathematics departments go to great efforts to state prerequisites for a course, usually check to determine if a student has mastered all prerequisite material, and are usually insistent that no student enroll in a course without the necessary prerequisites. That obsession with prerequisites results from a clear understanding that a mathematics concept cannot be learned if it depends upon un-mastered prerequisite material.

It is to be expected that occasional prerequisite material will be unknown to the learner. When such unknown prerequisite material is identified the learner is expected to interrupt the current instructional activity long enough to learn the necessary prerequisite material. When the amount of such unknown prerequisite materials is excessive the amount of time required becomes excessive and the learn is unable to succeed.

(b) Study time

The amount of time required to learn a particular mathematics topic varies from individual to individual. Mathematics departments across the country are fond of recommending two hours of study time outside of class for every hour of in-class lecture. My personal experience indicates that this is a minimal requirement. Many students need as much as three or four hours of study time outside of class for every hour of in-class instruction. A few individuals will require many more hours of instruction and study in order to succeed. There is a brutal fact embedded in the previous sentences. If you

cannot (or do not) devote the amount of time required for you to learn the mathematics presented in College Algebra (or any other math course), you will fail the course.

In many instances the amount of required study time is extended because necessary prerequisite material has not been mastered. When prerequisite material has not previously been mastered, the learner must interrupt his/her study of the current topic to study and master that necessary prerequisite material before attempting to proceed. For example when a learner attempts to understand the definition of complex number, if the symbol $\sqrt{-1}$ has not previously been mastered, the learner must interrupt the study of this definition while finding and mastering definitions and explanations of square root and principle square root. Obviously that activity requires additional time.

(c) Study methods

Some students learn better if the instructional stimulus is written while others learn better if the instructional stimulus is audio. There are numerous other individual learner preferences of this nature, none of which will be discussed here.

What constitutes appropriate study methods for learning mathematics is less dependent on the individual than on the sequence of activities and desired learning outcomes. I will discuss a necessary sequence of learner activities and the expected learning outcomes from each of these learner activities.

Nationally more than 50% of students in elementary mathematics classes fail mainly because they do not properly study mathematics. National statistics indicate that about 60% of beginning mathematics students need to change study habits.

Learner Activity 1: Memorize definitions

The very first and absolutely essential step to learning a mathematics topic is to memorize the definitions involved. As an example suppose the topic to be learned is the concept of equality of complex numbers. Begin by memorizing:

Two complex numbers a + bi and c + di are equal
a + bi = c + di if and only if a = c and b = d

At the end of this activity the learner should be able to write the definition without errors.

Learner Activity 2: Study and understand definitions

Decide what the definition says. The above definition states that two complex numbers are equal if and only if their corresponding real components are equal and their corresponding complex components are equal.

Make note of each symbol and be sure you understand its role in the definition. In the above definition the blue = symbol means equality of complex numbers (the new concept) and the red = symbols mean equality of real numbers (an old familiar concept). If the old concept is not familiar, you must go back and find and review its meaning in previously learned material.

At the end of this activity the learner should be able to explain each mathematical word and symbol used in the definition.

Learner Activity 3: Construct questions and answers about each definition
Ask yourself if you know the definition of each mathematical word or symbol used in the new definition. A good way to formulate that question to yourself is: Write the definition of ----.
For example, when studying the definition of equality of complex numbers you should ask yourself the following questions:

Write the definition of complex number.
Write the definition of real component.
Write the definition of complex component.
Write the definition of i.
Does this define a concept or a mathematical object?
What is the value of i^2 ?
What does the blue = mean?
What do the red = symbols mean?

You should answer these questions by writing in complete sentences. When writing a definition insist that it be absolutely letter perfect. When any of us studies a new topic we are not knowledgeable enough to rephrase a definition.

At the end of this activity the learner should be able to write a precise definition for each mathematical word and symbol used in the definition. The learner should be able to intelligently discuss the use of each word and symbol in this definition.

Learner Activity 4: Construct a list of examples and a list of non-examples
Things that satisfy the definition are examples of the creature being defined. Things that do not satisfy the definition are non-examples of the creature being defined. You should construct lists of things that satisfy the definition and things that do not satisfy the definition. For example, when studying the definition of a complex number you might construct the following lists.

Examples		Non-Examples
$3+4i$	$4+\sqrt{-5}$	$3 + 2x$
$-5+\frac{3}{4}i$	5i	$\begin{bmatrix} 3 & 5 \\ 0 & -2 \end{bmatrix}$
$6-\sqrt{3}i$	8	$2+5x=3x-6$
$(a-1)+(b+3)i$		

At the end of this activity the learner should be able to correctly identify examples and non-examples for the concept being defined.

Learner Activity 5: Evaluate your understanding by attempting some exercises

Use the exercises at the end of the section to evaluate your understanding of the concept. For the above definition turn to exercises like:

Find real numbers a and b such that $(a - 1) + (b + 3)i = 5 + 8i$. Analyze the question by observing the equal sign = refers to equality of complex numbers and therefore the above definition must apply. From that definition you can then conclude that $a - 1 = 5$ and $b + 3 = 8$. Solving those two equations for a and b provides a nice simple review of one of the properties of equations.

If you are unable to answer the question, you must conclude that you have not mastered the definition of equality of complex numbers and you must work through the first four activities again. More carefully redoing the first four activities may clear up the difficulty but it may be necessary to redo them with some assistance from an instructor or tutor.

> **Comment:**
> Observe that working the problems should occur rather late in the study process. Significant study should occur before working problems.
>
> Mathematics is not learned by working problems. Mathematics is learned in order to solve problems. The primary purpose for exercises in mathematics textbooks is to provide a means of self evaluation.

Learner Activity 6: Study available examples, illustrations, and applications

Go through the text material, website material, and lecture notes to find all examples relevant to this definition. Study each very carefully. To study very carefully means that you must know and understand every word and symbol being used. If an unfamiliar word or symbol is used, you must go back and find and review its meaning in previously learned material.

Learner Activity 7: Evaluate your understanding by attempting some exercises

The examples, illustrations and applications frequently will extend the concept beyond simple understanding of the concept. In other instances examples will illustrate how two or more concepts may be used in concert to solve a problem or answer a question. To evaluate how well you have mastered the extensions, you should turn to related exercises at the end of the section.

For example, when studying the definition of complex number, at this stage you might want try exercises like: "Write the complex number $4i + i^2$ in standard form." or "Write the complex number $1 + \sqrt{-18}$ in standard form." If you can correctly perform those activities and provide sound mathematical reason for your process, then you should probably conclude that you have mastered the definition of complex number, the definition of $\sqrt{-1}$, and the definition of the principle square root of a negative real number. If you cannot answer those questions or if

you cannot provide reasons for your work, then you should reexamine the definitions, examples, illustrations, and any other explanations you have at your disposal.

Learner Activity 8: Periodic Review
Implement a schedule for regularly (every week) reviewing definitions and their ramifications. When performing your periodic review also try to consider all subsequent uses of the concept.

For periodic review you might consider a set of flash cards containing each of the definitions in the course. Another method which has been successfully used by many persons (myself included) is to maintain a notebook containing each definition ever encountered in mathematics. Regularly read a part of this notebook. For example, devote 15 minutes each day to reading in this notebook.

Learner Activity 9: Study Properties, Procedures, etc
To study any special properties and or procedures, you should attack each one individually in the same manner that you studied the definitions.

Learner Activity 10: Evaluate your understanding by attempting some exercises
To evaluate how well you have mastered the additional properties and procedures, you should turn to related exercises at the end of the section. For example, after mastering the concepts of norm of a complex number, and conjugate of a complex number, and multiplication of complex numbers and after studying the procedure for converting a division problem to a multiplication problem, you should turn to some division problems at the end of the section. If you can perform the divisions correctly and if you can provide reasons for each step in your work, then you should conclude that you are ready do move on to the next topic. If you cannot correctly perform the divisions or if you cannot provide correct and complete deductive reasoning for your work, then additional study is required.

Learner Activity 11: Understand relations between concepts
Read your notes, text, and website looking for statements that relate concepts to each other. Try to understand how the various concepts fit together to constitute a whole unified structure. Try to answer question like:

How do the new concepts relate to familiar concepts.
Are they similar ?
Are they the same?
Are they in opposition?
How does the new concept extend your ability to use mathematics?

For example, what are the similarities and differences between division of fractions and division of complex numbers?

At this stage of your development you should be able to write intelligently about the topics. For example you should be able to write a paragraph or two about the

similarities of division of fractions and division of complex numbers. You should be able to write each definition, concept, property, and procedure without any prompting.

Learner Activity 12: Evaluate your understanding of larger blocks of material

Use the exercises at the end of the section or chapter to evaluate your understanding of collection of concepts contained in the section or chapter. Use the more difficult exercises and/or self-test for this evaluation. Test yourself by working old tests and/or quizzes from the website.

At this stage of your development you should be able to present a correct lecture on the topics of the section and/or chapter. You should also be able to construct tests and quizzes which test all levels of knowledge of the topics.

2) How should I study for your tests.

You should not study for my tests. You should study to learn the mathematics. If you know the mathematics, you will do well on my tests.

You may look at tests and quizzes which I have given in past semesters. Examining those old tests might avoid surprises with respect to the type of question. Those old tests are available on the website

You cannot train for my tests. By that I mean you cannot practice a collection of exercises and expect the test to be questions like the ones you practiced. My tests are designed to determine if you know the concepts and can use those concepts.

3) How do you want me to write the answer.

The short answer is: Correctly

a) Write correct mathematics in correct complete sentences.

Look at the list of common errors which I have handed out. Look at the common errors on the website under special topics. Look at the critique of tests on the website. Avoid all the error types enumerated in these lists.

Study and try to imitate the writing styles used in the textbook, on the website, in the lectures, on old test solutions, and all other forms of approved instructional material.

b) Answer the question that is asked.

Think about what you write. Your response should not be some conditioned response elicited by the form of a mathematical expression in the question. Here are some examples.

If the question requests you to write the equation obtained by adding $3x - 1$ to both sides of the equation $x + 9 = -3x + 1$

your response should be $4x + 8 = 0$. It should not be $x = -2$.

If the question is: What is the solution set for the equation $x^2 + 5x + 6 = 0$

Your response should include all the necessary logic to solve the equation and your conclusion should be:

"The solution set for the equation $x^2 + 5x + 6 = 0$ is { -3, -2 }"

If the question is: What are the solutions of the equation $x^2 + 5x + 6 = 0$

Your response should include all the necessary logic to solve the equation and your conclusion should be:

"The solutions for the equation $x^2 + 5x + 6 = 0$ are -3 and -2 "

If the question is: Solve the equation $x^2 + 5x + 6 = 0$

Your response should include all the necessary logic to solve the equation and your conclusion should be:

"The solution set for the equation $x^2 + 5x + 6 = 0$ is { -3, -2 }" because to solve an equation means to find the solution set for the equation. However, because textbooks traditionally have been lax on this issue, when simply asked to solve an equation,

Your response should include all the necessary logic to solve the equation and your conclusion may be:

"The solutions for the equation $x^2 + 5x + 6 = 0$ are -3 and -2 "

If the question requests that your answer be expressed in a certain form such as interval notation, set builder notation for a set, roster notation for a set, graphical representation, fraction reduced to lowest terms, or a decimal correct to three decimal places, etc., then your response must adhere to those instructions.

If the instructions in a question do not request a particular form, then any correct answer will be considered to be correct. All responses must correctly use complete sentences containing standard mathematics notation and writing conventions.

c) Convince me

One purpose for a test is to help me assess your mastery of the subject. Therefore your response to a test question should convince me that you know and understand the concept(s) addressed in that question. For example, if the question is about solving an equation, I expect your response to be evidence that you understand the concepts involved in solving an equation.

I am rarely interested in your arithmetic prowess. Mathematics and computation are not synonyms. Computation is a tiny part of mathematics and most computation is now relegated to calculators. Show me your understanding of concepts, intellectual skills, and knowledge of processes.

Your response should convince me that you understand the concept in question. Remember that finding the right answer is no more evidence that you understand mathematics than finding an acorn is evidence that you can identify an oak tree.

d) Permit me to assign a good grade
Write your work in a manner that permits me to assign a good grade to your performance.

As indicated in an earlier document distributed to this class.
In this course you will be graded on:

1. Your ability to correctly state mathematical concepts.
2. Your ability to correctly use mathematical terms and symbols when writing.
3. Your ability to correctly use mathematical concepts to solve problems.
4. The correctness of your method for solving a problem.
5. Your written presentation of a process for solving a problem.
6. Your understanding of mathematical concepts as exemplified by your writing.
7. Your ability to recognize and use connections within mathematics.
8. Your ability to formulate and use generalizations.