

Writing Mathematics Correctly

Guidelines for Math 160C

by
Delano P. Wegener, Ph.D.
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In this course you will be graded on:

1. Your ability to correctly state mathematical concepts.
2. Your ability to correctly use mathematical terms and symbols when writing.
3. Your ability to correctly use mathematical concepts to solve problems.
4. The correctness of your method for solving a problem.
5. Your written presentation of a process for solving a problem.
6. Your understanding of mathematical concepts as exemplified by your writing.
7. Your ability to recognize and use connections within mathematics.
8. Your ability to formulate and use generalizations.

Even a casual reading of the above eight items reveals that I will place considerable emphasis on writing mathematics correctly. Your previous experience in mathematics may well lead you to believe that such emphasis on writing in mathematics is an aberration. I have written this brief essay to remove all doubt about the need and value of writing in mathematics. It is based on quotations from respected authorities and educators as well as my personal observations during more than 30 years of teaching.

You might also observe from the above that you will not be graded on writing down some final “answer”.

“There is good reason why Herman Melville wrote *Moby Dick* as a novel and not as the single sentence:

The whale wins.

For the same reason, just writing down your final conclusions in an assignment will not be sufficient in a college math class.” [Lee]

Professor Lee also correctly claims;

“The ideas are the mathematics. So a page of computations without any writing or explanations contains no mathematics.” [Lee]

Establishing the Need for Good Writing

Included in the Missouri State Level Goals for General Education is the following statement about mathematics.

“Students should develop a level of quantitative literacy that would enable them to make decisions and solve problems and which could serve as a basis for continued learning.”

The following statements are in part the intended implementation, by STLCC, of the State Level General Education Goals.

“Represent mathematical information graphically, symbolically, numerically, and verbally with clarity, accuracy, and precision.”

“Formulate and use generalizations based upon pattern recognition.”

“Recognize and use the connections within mathematics.”

In 1992, the state of Tennessee State Board of Education set forth a number of goals related to mathematics for all high school students. Included in that list of five goals was:

“Learn to communicate mathematically.”

In the following year that same board made a number of specific recommendations to be implemented in the mathematics curriculum. First in that list of goals was to:

“Read, write, and orally communicate mathematical concepts.”

Many other states have similar policies regarding communication and mathematics.

We may conclude from the above and numerous other sources that:

Educators at the state level are in agreement that mathematical communication is important.

Individual educators share this view. Dr. Kevin P. Lee provides a good example: [Lee]

“The mathematics learned in college will include concepts which cannot be expressed using just equations and formulas.”

“... being able to write clearly is as important a mathematical skill as being able to solve equations.”

Why is Writing Important in Mathematics?

In a statement of his teaching philosophy, Professor Maurer of Swarthmore College states:

“Writing is an essential form of communication, especially for subtle material like mathematics. Some people think writing and mathematics are disjoint activities, but far from it. In mathematics you use all the tools of ordinary language plus the additional conventions of mathematical symbolism – solutions consist of both words and symbols. So writing plays an important role in my courses.” [Maurer2]

In the first paragraph of a 1996 essay, E. Berry and J. Lawson state:

“In any discipline, the successful communication of ideas is at least as important as the ideas themselves. Most disciplines develop standard usages and restrictions that differ from everyday English. Mathematics is not an exception.” [Berry]

There is widespread agreement among educators that writing mathematics helps students learn the concepts. There is also almost universal agreement that communication in the discipline is essential to utilizing any discipline in everyday life, and that good communication skills are important to career advancement..

Methods to Improve Mathematics Writing in This Course

The role of definitions in mathematical writing and the proper form for writing definitions will be emphasized through a number of assignment activities which will require you to write important mathematical definitions. Absolute perfection will be demanded for these assignments.

A number of assignment activities will contain a model for writing a particular type of solution process. You will then be expected to adhere to that model to write solutions for several exercises.

The statement of Quiz and Test problems may contain the final “answer” and request that you write a proper argument that leads to the given conclusion.

Examples in the textbook illustrate proper mathematics writing.

Basic Rules for Writing Mathematics

“Mathematics writing is different from ordinary writing and harder — in addition to all the requirement of ordinary good writing, there are additional constraints and conventions in mathematics. An additional constraint is that mathematics follows much more demanding rules of logic than ordinary discourse, and you must make your logic clear. Some of the additional conventions are those for defining new concepts and those for organizing the material “[Maurer, p.3]

There are two important aspects to writing mathematics correctly:

- 1) The mathematics must be correct.
- 2) The writing must be grammatically correct.

Some basic rules for writing in general and writing mathematics in particular are presented here. More rules and conventions will be presented as the semester progresses. No significance should be attached to the order in which these rules are presented.

“The fact that some mathematics conventions have been universally adopted around the world suggests that they accomplish something important.” [Maurer] For that reason it is unwise for a novice to vary from standard conventions, even though they are not absolute rules.

- Write with a pencil so that errors may be erased and replaced with correct work.
- Use complete sentences, correct grammar, and correct spelling.
- Symbols (like the $+$ symbol) that have a specific mathematical meaning are reserved for mathematical use.
- Don't submit your first draft.
- Many mathematical adjectives and nouns have precise mathematical meanings, and an English synonym will not serve as a replacement. For example, "element" and "part" are not interchangeable when referring to an element of a set.
- Look at examples of writing in the textbook, and try to emulate the style.
- Start each sentence with a word, not a mathematical symbol.
- Two mathematical expressions or formulas in a sentence should be separated by more than just a space or by punctuation; use at least one word.
- Words have meanings: be aware of them. For example, an equation has an equal sign in it.
- Don't use abbreviations.
- Don't end a line with an equal sign or an inequality sign.
- Insure that every statement is mathematically correct.
- Strive for a good balance between words and symbols.
- Honor the equal sign.
- Use different letters for different things.
- Remember mathematics is case sensitive.
- Define any terms or variables which you use.
- Once a variable has been assigned a meaning, do not re-use it with a different meaning in the same context.
- There is a distinction between the definite article ("the") and the indefinite articles ("a" and "an").
- Avoid the use of imprecise terms.
- Be sure that your use of a term agrees with the definition of that term.
- Include transitional phrases to help guide the reader.
- Conclude the solution of a problem with a clear and complete statement of your conclusion.

Some of the above come from: "Writing Mathematics"[Berry] and some from "Course Description for Math 248 at The University of Illinois" [Grayson].

Common Errors to Avoid When Writing Mathematics

The most common errors fall into the following categories:

- not answering the question
- incorrect English
- incorrect mathematics
- hard-to-read format
- use of pronouns such as it
- using the same symbol for different quantities
- too many words and too few symbols, or vice versa
- leaving too little space for possible corrections

Listed here are some of the common errors which you should avoid. This list is certainly not complete. Additions will be made to it in future semesters and a supplement or duplicate list will be maintained on the DrDelMath website.

(http://www.drdelmath.com/special_topics/common_errors.htm)

Don't write $f = x + 1$	when you mean	$f(x) = x + 1$
Don't write $n = \text{even} = 2n$	when you mean	If n is even, then $n = 2k$ for some k
Don't write $n^2 = 16 = n = \pm 4$	when you mean	$n^2 = 16$ implies $n = \pm 4$
Don't write $k = k + 1$	when you mean	Replace k by $k + 1$
Do not write $(3, 4, 8)$	when you mean	$\{3, 4, 8\}$
Do not write $a \subset A$	when you mean	$a \in A$
Don't write length + area	when you mean	length and area
Don't write 1.4	when you mean	$\sqrt{2}$

Do not confuse the words equation, expression, and function. An expression is an algebraic combination of terms containing no verb. An equation is a mathematical statement which contains an equal sign. A function consists of a rule, a domain, and a range.

It is incorrect to speak or write of the quotient of a and b . When speaking or writing about division, your statements must make it clear which is the divisor and which is the dividend.

It is correct to write:	divide both sides of the equation by 10
It is incorrect to write:	divide 10 to both sides of the equation
It is incorrect to write:	divide 10 by both sides of the equation
It is incorrect to write (or do)	divide the right side of the equation by 10

eg. Faced with $10x = 20$ it is incorrect to claim that dividing 20 by 10 yields $x = 2$.

It is incorrect to speak or write of the difference of a and b . When speaking or writing about subtraction, your statements must make it clear which is the subtrahend and which is the minuend.

It is correct to write:	subtract $5x$ from both sides of the equation
It is incorrect to write:	subtract $5x$ to both sides of the equation
It is incorrect to write:	subtract both sides by $5x$
It is incorrect to write:	minus $5x$ from both sides of the equation
It is incorrect to write:	$- 5x$ from both sides of the equation

It is incorrect to write or speak of moving variables or numbers from one side of the equation to the other.

It is incorrect to write $3 + - 4$.	Correct syntax is $3 + (- 4)$
It is incorrect to write $3 \frac{+}{-} - 4$.	Correct syntax is $3 \frac{+}{-}(- 4)$
It is incorrect to write $3 - - 4$.	Correct syntax is $3 - (- 4)$

References:

- [Lee] "A Guide to Writing Mathematics"
(<http://ems.calumet.purdue.edu/mcss/kevinlee/mathwriting/writingman.pdf>)
- [Lee2] "Tips for Reading Mathematics"
(<http://ems.calumet.purdue.edu/mcss/kevinlee/mathwriting/readingtips.pdf>)
- [Lee3] "A Mathematical Writing Checklist"
(<http://ems.calumet.purdue.edu/mcss/kevinlee/mathwriting/writingcheck.pdf>)
- [Maurer] "Advice for Undergraduates on Special Aspects of Writing Mathematics"
(http://www.swarthmore.edu/library/cornell/WRITE_PRIMUS.pdf)
- [Berry] "Writing Mathematics"
(<http://www.trinity.edu/jlawson/teach/writing/writing.pdf>)
- [Grayson] "Course Description for Math 248 at The University of Illinois"
(<http://www.math.uiuc.edu/~dan/Courses/2003/Fall/248/>)
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