**Meramec Intermediate Algebra TEST 4 Solution Summer 2009**

NAME: ________________________________ Score____________________________/100

Please print

**SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION**

2 points each for questions 1 – 10. 7 points each of the other questions.

**Circle T or F, whichever is correct.**

1. **T** F Two rational expressions must have the same denominator before they can be added.

2. T **F** Two rational expressions must have the same denominator before they can be multiplied.

3. T **F** If both sides of an equation are multiplied by an expression containing a variable, the resulting equation will be equivalent to the original equation.

4. **T** F A fraction has been reduced if the numerator and denominator have no common factors other than 1.

5. **T** F Unless otherwise stated the domain of a rational expression is the largest set of real numbers for which the expression makes sense.

6. **T** F If a represents a real number and b and c represent non-zero real numbers then $\frac{ac}{bc} = \frac{a}{b}$

7. **T** F $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b} = -\frac{-a}{-b}$

8. T **F** $\frac{\frac{3}{4}}{\frac{5}{7}} = \left(\frac{3}{4}\right)\left(\frac{5}{7}\right)$

9. **T** F Every subtraction problem can be changed to an addition problem.

10. **T** F Every division problem can be changed to a multiplication problem.

**Perform the indicated operations. Simplify the results as much as possible.** **HINT: Watch for cancellations.**

**In an attempt to further explain "cancellation" I will include more detail in the following solutions than might normally be required.**

11. $\left(\frac{2x-4}{15}\right)\left(\frac{6}{2-x}\right)$

$$\left(\frac{2x-4}{15}\right)\left(\frac{6}{2-x}\right) = \frac{(2)(x-2)(6)}{(15)(-1)(x-2)} = \frac{(2)(2)(3)(x-2)}{(3)(5)(-1)(x-2)} = \left(\frac{\cancel{3}}{\cancel{3}}\right)\left(\frac{\cancel{x-2}}{\cancel{x-2}}\right)\left(\frac{(2)(2)}{(-1)(5)}\right) = -\frac{4}{5}$$

12. $\left(\frac{2x^2-2}{10x+30}\right)\left(\frac{12x+36}{3x-3}\right)$

$$\left(\frac{2x^2-2}{10x+30}\right)\left(\frac{12x+36}{3x-3}\right) = \frac{(2)(x^2-1)(12)(x+3)}{(10)(x+3)(3)(x-1)} = \frac{(2)(2)(2)(3)(x-1)(x+1)}{(2)(5)(3)(x-1)(x+3)}$$

$$= \left(\frac{\cancel{2}}{\cancel{2}}\right)\left(\frac{\cancel{3}}{\cancel{3}}\right)\left(\frac{\cancel{x-1}}{\cancel{x-1}}\right)\left(\frac{\cancel{x+3}}{\cancel{x+3}}\right)\left(\frac{4(x+1)}{5}\right) = \frac{4(x+1)}{5}$$

13. $\left(\frac{x^2-6x+9}{x^2-x-6}\right)\div\left(\frac{x^2-9}{4}\right)$

$$\left(\frac{x^2-6x+9}{x^2-x-6}\right)\div\left(\frac{x^2-9}{4}\right)=\left(\frac{x^2-6x+9}{x^2-x-6}\right)\left(\frac{4}{x^2-9}\right)=\frac{(x-3)^2 4}{(x-3)(x+2)(x-3)(x+3)}$$

$$=\left(\frac{(x-3)^2}{(x-3)^2}\right)\left(\frac{4}{(x+2)(x+3)}\right)=\frac{4}{(x+2)(x+3)}$$

14. $\left(\frac{x-3}{x+4}\right)+\left(\frac{x+2}{x-4}\right)$

$$\left(\frac{x-3}{x+4}\right)+\left(\frac{x+2}{x-4}\right)=\frac{(x-3)(x-4)}{(x+4)(x-4)}+\frac{(x+2)(x+4)}{(x-4)(x+4)}=\frac{x^2-7x+12}{(x-4)(x+4)}+\frac{x^2+6x+8}{(x-4)(x+4)}$$

$$=\frac{\left(x^2-7x+12\right)+\left(x^2+6x+8\right)}{(x-4)(x+4)}=\frac{2x^2-x+20}{(x-4)(x+4)}$$

$2x^2-x+20$ cannot be factored.

15. $\left(\frac{x-3}{x-4}\right)-\left(\frac{x+2}{4-x}\right)$

**To begin we note that all subtractions are performed by changing to addition by adding the opposite of the subtrahend. You should observe that a useful way of writing the opposite of** $\frac{x+2}{4-x}$ is $\frac{x+2}{x-4}$**.**

**Then we can proceed as follows.**

$$\left(\frac{x-3}{x-4}\right)-\left(\frac{x+2}{4-x}\right)=\left(\frac{x-3}{x-4}\right)+\left(\frac{x+2}{x-4}\right)=\frac{(x-3)+(x+2)}{x-4}=\frac{2x-1}{x-4}$$

16. $\left(\frac{9x+2}{3x^2-2x-8}\right)+\left(\frac{7}{3x^2+x-4}\right)$.

$$\left(\frac{9x+2}{3x^2-2x-8}\right)+\left(\frac{7}{3x^2+x-4}\right)=\frac{9x+2}{(3x+4)(x-2)}+\frac{7}{(3x+4)(x-1)}=\frac{(9x+2)(x-1)}{(3x+4)(x-1)(x-2)}+\frac{7(x-2)}{(3x+4)(x-1)(x-}$$

$$=\frac{\left(9x^2-7x-2\right)+(7x-14)}{(3x+4)(x-1)(x-2}=\frac{9x^2-16}{(3x+4)(x-1)(x-2}=\frac{(3x+4)(3x-4)}{(3x+4)(x-1)(x-2)}=\left(\frac{3x+4}{3x+4}\right)\left(\frac{3x-4}{(x-1)(x-2)}\right)$$

$$=\frac{3x-4}{(x-1)(x-2)}$$

17. $\left(\frac{17x+4}{4x}\right)-\left(\frac{17x-4}{4x}\right)$.

$$\left(\frac{17x+4}{4x}\right)-\left(\frac{17x-4}{4x}\right)=\left(\frac{17x+4}{4x}\right)+\left(\frac{4-17x}{4x}\right)=\frac{8}{4x}=\frac{2}{x}$$

18. Simplify $\left(\dfrac{\frac{x+5}{x}}{\frac{x+5}{x}+2}\right)$.

$$\left(\dfrac{\frac{x+5}{x}}{\frac{x+5}{x}+2}\right) = \left(\dfrac{\frac{x+5}{x}}{\frac{x+5}{x}+\frac{2x}{x}}\right) = \left(\dfrac{\frac{x+5}{x}}{\frac{x+5+2x}{x}}\right) = \left(\dfrac{\frac{x+5}{x}}{\frac{3x+5}{x}}\right) = \left(\frac{x+5}{x}\right)\left(\frac{x}{3x+5}\right) = \left(\frac{\cancel{x}}{\cancel{x}}\right)\left(\frac{x+5}{3x+5}\right) = \frac{x+5}{3x+5}$$

19. What is the domain of the rational expression $\dfrac{x^2+5x+5}{x+6}$

By convention, the domain is all real numbers for which the expression makes sense (is defined). This expression is defined for all real numbers except those which make the denominator 0. Therefore the domain of this expression is all real numbers except - 6. If we want to use notations to write this we can write:

$(-\infty,-6)\cup(-6,\infty)$ or $\{x \mid x \neq -6\}$

20. Find the solution set of the equation $\dfrac{2x+1}{4-x} = \dfrac{9}{4-x}$.

Multiply both sides of the equation by 4 – x to obtain

2x + 1 = 9 (which might not be equivalent to the original equation)

Add – 1 to both sides of the equation

2x = 8

Multiply both sides by ½

x= 4 Note that 4 is the only possible solution, but it is only a possibility.

Testing 4 in the original equation produces a 0 in a denominator.

Therefore 4 is not a solution of the original equation and now it follows that

The solution set for the original equation is the empty set ∅

21. Find the solution set of the equation $\dfrac{36}{x^2-9}+1 = \dfrac{2x}{x+3}$

Multiply both sides of the equation to obtain

$36 + (x^2 – 9) = (2x)(x – 3) = 2x^2 – 6x$

Add the opposite of $36 + x^2 – 9$ to both sides to obtain

$x^2 – 6x – 27 = 0$

(x – 9)(x + 3) = 0 And then from the Zero Factor Property we conclude

x = 9 or x = – 3

9 and – 3 are the only possible solution of the original equation. However, – 3 causes a 0 in a denominator and is therefore not a solution. On the other hand, 9 does not cause a 0 in any denominator and is therefore a solution.

The solution set for the original equation is {9}.