Meramec Intermediate Algebra TEST 2 Spring 2010

NAME: _____ Score_____/100
Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct. (2 pts each)

1. T F The graph of a linear inequality in one variable is a dot on the real number line.

2. T F If any expression is added to both sides of an equation the resulting equation is equivalent to the original equation.

3. T F If any real number is added to both sides of an equation the resulting equation is equivalent to the original equation.

4. T F If both sides of an equation are multiplied by the same expression, the resulting equation is equivalent to the original equation.

5. T F The graph of a conditional linear equation in one variable is a dot on the real number line .

6. T F The graph of an equation or inequality in one variable is a subset of the real number line.

7. T F The solution set for an equation is the set of all solutions for that equation.

8. T F The equation $3x + 2 = -7x + \pi$ is a linear equation in one variable.

9. T F Two inequalities are equivalent inequalities if they have the same solution sets.

Fill in each of the blanks to make the statements true. (2 pts each)

10. A conditional equation is an equation which is true when some real numbers are substituted for the variables and is false when some real numbers are substituted for the variables.

11. The graph of an inequality consists of all the points, and only those points, whose coordinates are solutions of the inequality.

12. Two equations are equivalent if they have the same solution sets.

13. If both sides of an inequality are multiplied by the same negative real number and the inequality symbol is reversed, the resulting inequality is equivalent to the original inequality.

14. The formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 h$.

15. The formula for the volume of a rectangular solid of length a, width b, and height c is $V = abc$.

16. The process to solve a linear equation in one variable is to generate a sequence of equations each equivalent to the previous equation until a simplest equation is obtained.

17. If 7 is not a solution of $x^4 - x^2 + x + 1 > 2$ and 7 is not a solution of $x^4 - x^2 + x + 1 < 2$, then 7 is a solution of $x^4 - x^2 + x + 1 = 2$.

18. Complete the statement of the Law of Trichotomy.

If a and b are real numbers, then one and only one of the following is true:

i) $a < b$

ii) $a = b$

iii) $a > b$

19. If a true statement results when 23 is substituted for the variable in an equation, then 23 is a solution of the equation.

20. The graph of an equation is the boundary between the graphs of the corresponding two inequalities

21. **(5 points)** Label, by circling the correct word, each of the following as an expression, equation, or inequality.

a) $\frac{1}{3}x - 5 = 6$ (expression equation inequality)

b) $2(x - 3) > 7$ (expression equation inequality)

c) $x + 4x^3 - 1$ (expression equation inequality)

d) $\frac{5}{9}x + \frac{1}{3} < \frac{2}{9} - \sqrt{17}x$ (expression equation inequality)

e) $\frac{5}{9}x - \frac{2}{3} + \frac{2}{9} - \sqrt{17}x^2$ (expression equation inequality)

22. **(5 points)** Label, by circling the correct word, each of the following as an identity, a conditional equation, or a contradiction.

a) $5x + 3 = 3 + 5x$ (identity conditional contradiction)

b) $2x + 1 = 2x - 3$ (identity conditional contradiction)

c) $5x - 2 = -7 + 5x$ (identity conditional contradiction)

d) $5x - 3 = 2x + 3$ (identity conditional contradiction)

e) $x^2 = -5$ (identity conditional contradiction)

23. **(5 points)** Which of the following are linear equations in one variable. Indicate your answer by circling YES or NO.

a) $2x - 4 = 7y$ (YES NO)

b) $\frac{5x}{2} = 6x$ (YES NO)

c) $2x + 3 = 12$ (YES NO)

d) $\pi x + \sqrt{3} = \frac{15}{4}x - 2$ (YES NO)

e) $x^2 - 2x + 7 = 2$ (YES NO)

Show your work on exercises 24 – 32 inclusive (5 pts each). **No work –No Credit Be neat! Include enough words so I can understand what you are doing. Be organized.**

24. The volume of a cone with radius $\frac{1}{2}$ is $\frac{2}{3}\pi$. What is the height of the cone?

Use $V = \frac{1}{3}\pi r^2 h$ to obtain

$$\frac{2}{3}\pi = \frac{1}{3}\pi\left(\frac{1}{2}\right)^2 h$$

Then simplfy $\frac{2}{3}\pi = \frac{1}{3}\pi\left(\frac{1}{2}\right)^2 h = \frac{1}{3}\pi\frac{1}{4}h = \frac{\pi}{12}h$

Solve for h by multiplying both sides by $\frac{12}{\pi}$

$$h = \left(\frac{2\pi}{3}\right)\left(\frac{12}{\pi}\right) = 8$$

The cone is 8 units high.

25. The solution set for $x^3 + x^2 + x + 1 = 0$ is $\{-1\}$.

a) What is the solution set for $x^3 + x^2 + x + 1 < 0$?
Test 0 in the inequality $x^3 + x^2 + x + 1 < 0$ to obtain $1 < 0$ which is false.
Therefore the solution set for $x^3 + x^2 + x + 1 < 0$ is everything on the other side of -1.
The solution set for $x^3 + x^2 + x + 1 < 0$ is $(-\infty,-1)$.

b) What is the solution set for $x^3 + x^2 + x + 1 > 0$?
According to Part a,
the solution set for $x^3 + x^2 + x + 1 = 0$ is $\{-1\}$ and
the solution set for $x^3 + x^2 + x + 1 < 0$ is $(-\infty, -1)$.
According to the Law of Trichotomy,
the solution set for $x^3 + x^2 + x + 1 > 0$ is everything else.
Therefore the solution set for $x^3 + x^2 + x + 1 > 0$ is $(-1, \infty)$.

26. Solve $T = C(2 + AB)$ for B

Use the Distributive property to perform the muliplication
$T = 2C + ABC$ add $-2C$ to both sides

$T - 2C = ABC$ multiply both sides by $\frac{1}{AC}$

$$B = \frac{T - 2C}{AC}$$

27. Solve the equation $5x + 3 = 11x + 9$. Reduce all fractions in your answer. Don't use mixed numbers. Don't use decimals. Improper fractions are just fine. Irrational numbers are just fine.

$5x + 3 = 11x + 9$

$-6x + 3 = 9$

$-6x = 6$

$x = -1$

The solution set is $\{-1\}$.

28. Solve the equation $5x + \sqrt{5} = x + 1$. Reduce all fractions in your answer. Don't use mixed numbers. Don't use decimals. Improper fractions are just fine. Irrational numbers are just fine.

$5x + \sqrt{5} = x + 1$

$4x + \sqrt{5} = 1$

$4x = 1 - \sqrt{5}$

$x = \dfrac{1 - \sqrt{5}}{4}$

The solution set is $\left\{\dfrac{1 - \sqrt{5}}{4}\right\}$.

29. Solve the inequality $2x - 7 < 7x + 2$. Write the solution set in both interval notation and set builder notation. Reduce all fractions in your answer. Don't use mixed numbers. Don't use decimals. Improper fractions are just fine. Irrational numbers are just fine.

Consider all three $2x - 7 < 7x + 2$, and $2x - 7 = 7x + 2$, and $2x - 7 > 7x + 2$
and solve the equality

$2x - 7 = 7x + 2$

$-7 = 5x + 2$

$-9 = 5x$

$x = \dfrac{-9}{5}$

Now test 0 in $2x - 7 < 7x + 2$ to get $-7 < 2$ which is true.

Therefore everything on that side of $\dfrac{-9}{5}$ is the solution set for $2x - 7 < 7x + 2$.

The solution set is $\left(-\dfrac{9}{5}, \infty\right) = \left\{x \middle| x > -\dfrac{9}{5}\right\}$

30. Solve and **graph** the inequality $2x - 4 > 8x + 1$. Reduce all fractions in your answer. Don't use mixed numbers. Don't use decimals. Improper fractions are just fine. Irrational numbers are just fine.

Consider all three $2x - 4 > 8x + 1$, and $2x - 4 = 8x + 1$, and $2x - 4 < 8x + 1$
and solve the equation.

$2x - 4 = 8x + 1$

$-4 = 6x + 1$

$-5 = 6x$

$x = -\frac{5}{6}$

Test 0 in $2x - 4 > 8x + 1$ to get $-4 > 1$ which is FALSE.

Therefore everything on the other side of $-\frac{5}{6}$ is the solution set for $2x - 4 > 8x + 1$.

The solution set for $2x - 4 > 8x + 1$ is $\left(-\infty, -\frac{5}{6}, \infty\right) = \left\{x \middle| x < -\frac{5}{6}\right\}$

In problems 31 and 32, complete the statements of the two basic properties of equations

31. If any expression is added to both sides of an equation the resulting equation is equivalent to the original equation.

32. If both sides of an equation are multiplied by the same non-zero real number the resulting equation is equivalent to the original equation.