

NAME: _____ Score _____ /100
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SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Questions 1 – 20 are 2 pts each.

1. **T** F The formula for the area of a triangle with base b and height h is $A = \frac{1}{2}bh$.
2. T **F** Every compound inequality can be written in compact form.
3. T **F** $3x + 7 > 2x - 9$ is an equation.
4. **T** F 5 is an element of $\{2, 9, 4, 3, 5\}$.
5. **T** F $2 < x < 9$ is a compact compound inequality.
6. **T** F The solution set for a conditional linear equation in one variable is a set containing a single number.
7. T **F** If both sides of an inequality are multiplied by the same negative real number the resulting inequality is equivalent to the original inequality.
8. T **F** The formula for the circumference of a circle with radius r is: $C = \pi r^2$.
9. **T** F The solution set for $|ax + b| < k$ is an interval on the number line.
10. **T** F The Law of Trichotomy informs us that each real number is a solution for one of $|ax + b| = k$, $|ax + b| < k$, or $|ax + b| > k$.
11. A **conditional** equation is an equation which is true when some real numbers are substituted for the variables and is false when some real numbers are substituted for the variables.
12. A number that makes an inequality **true** when substituted for the variable is called a solution of the inequality.
13. Two equations are **equivalent** if they have the same solution sets.
14. If any expression is added to both sides of an equation the resulting equation is **equivalent** to the original equation.
15. If both sides of an equation are multiplied by the same **non-zero real** number, the resulting equation is equivalent to the original equation.
16. The formula for the volume of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.
17. The distance d traveled at a rate r in time t is given by **$d = rt$** .
18. If an interval (p, q) is a solution to an inequality of the form $|ax + b| < k$, then the endpoints p and q are solutions of **$|ax + b| = k$** .

19. **Law of Trichotomy:** If a and b are real numbers, then one and only one of the following is true:

- i) $a < b$
- ii) $a = b$
- iii) $a > b$

20. The absolute value of a number is its distance from 0 on the number line. A more precise algebraic

definition is: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Questions 21 – 30 are 5 pts. each.

21. One garden is a square. A second garden is an equilateral triangle (sides are equal in length). The perimeter of the square garden is equal to the perimeter of the triangular garden. A side of the triangular garden is 15 feet longer than a side of the square. What are the dimensions of the gardens? **Hint: Draw pictures of each garden and label the sides. Then work with perimeter.**

Let P_s be the perimeter of the square.

Let P_t be the perimeter of the triangle.

Then $P_s = 4x$ and $P_t = 3(x + 15) = 3x + 45$

We are told $P_s = P_t$

Thus $4x = P_s = P_t = 3x + 45$ A double use of the Transitive Property yields

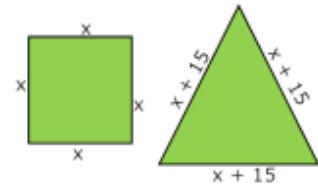
$4x = 3x + 45$ which is easily solved to produce

$x = 45$.

Each side of the square is 45 feet long.

Each side of the triangle is 60 feet long.

Notice that P_s and P_t both disappear from consideration when we get to the mathematical model.



22. On a recent day in St. Louis, the heat index was 104°F . Write that temperature in degrees Celsius. Use the formula $C = \frac{5}{9}(F - 32)$.

$$C = \frac{5}{9}(F - 32) = \frac{5}{9}(104 - 32) = \frac{5}{9}(72) = 5(8) = 40^\circ\text{C}$$

23. Solve $A = Prt + P$ for P .

$$A = Prt + P$$

$$A = P(rt + 1)$$

$$\frac{A}{rt + 1} = P$$

24. Write $(3, 8]$ in set-builder notation.

$$(3, 8] = \{x \mid 3 < x \leq 8\}$$

25. Write $\left\{x \mid \frac{3}{4} \leq x < 1\right\}$ in interval notation.

$$\left\{x \mid \frac{3}{4} \leq x < 1\right\} = \left[\frac{3}{4}, 1\right)$$

26. Solve $3x + 7 > 9x - 5$.

$$3x + 7 > 9x - 5$$

$$-6x + 7 > -5$$

$$-6x > -12$$

$$x < 2$$

The solution set for $3x + 7 > 9x - 5$ is $(-\infty, 2)$.

27. Show that 1 is a solution of the equation $5x^5 - 2x^4 = 3$.

$$5(1^5) - 2(1^4) = 3 \text{ is TRUE.}$$

**To show that a number is a solution of an equation you must show that it makes the equation TRUE!!!
If you did not make the observation that 1 made the equation true, you lost credit. This was not an exercise to see if you could subtract 2 from 5.**

28. Solve the equation $3(x - 6) = 5x$.

$$3(x - 6) = 5x$$

$$3x - 18 = 5x$$

$$-18 = 2x$$

$$x = -9$$

29. Solve the equation $3x + \sqrt{5} = \sqrt{2}x - \pi$.

$$\begin{aligned}3x + \sqrt{5} &= \sqrt{2}x - \pi \\3x - \sqrt{2}x &= -\sqrt{5} - \pi \\(3 - \sqrt{2})x &= -\sqrt{5} - \pi \\x &= \frac{-\sqrt{5} - \pi}{(3 - \sqrt{2})}\end{aligned}$$

30. The sum of two consecutive natural numbers is 525. Find the numbers.

Let n be the smaller of the two numbers. Then $n + 1$ is the other number.

We are told that

$$n + (n + 1) = 525$$

$$2n = 524$$

$$n = 262$$

$$\text{Then } n + 1 = 263.$$

The two numbers are 262 and 263.

31. (10 Pts.) Because $|3x - 6| < 9$ is equivalent to the compact compound inequality $-9 < 3x - 6 < 9$ we use computational methods to solve it.

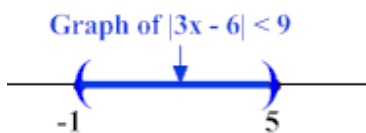
$$-9 < 3x - 6 < 9$$

$$-3 < 3x < 15$$

$$-1 < x < 5$$

The solution set for $|3x - 6| < 9$ is the interval $(-1, 5) = \{x | -1 < x < 5\}$.

The graph of $|3x - 6| < 9$ is



a) Based on the above work what is the solution set for $|3x - 6| = 9$? **Use the roster method to write the set.**

$$\{-1, 5\}$$

b) Based on the above work what is the solution set for $|3x - 6| > 9$? **Write the solution set in both set-builder notation and interval notation.**

$$(-\infty, -1) \cup (5, \infty) = \{x | x \leq -1 \text{ or } x > 5\}$$

c) Sketch the graph of $|3x - 6| > 9$.

