

Meramec Intermediate Algebra Sections 5.5-5.8 & Sections 6.1-6.3 TEST 5
Solution Summer 2010

NAME: _____ Score _____/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Course Average _____

No Decimals No mixed numbers No complex fractions No boxed or circled answers

Questions 1 – 20 are 2 pts each. Questions 21 – 35 are 4 pts each.

1. **T** F If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions, then their product is defined by $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right) = \frac{ac}{bd}$.
2. **T** F Division is done by changing to multiplication.
3. T **F** A fraction is an improper fraction if its numerator or its denominator (or both) contains a fraction.
4. **T** F A fraction has been reduced if the numerator and denominator have no common factors other than 1.
5. T **F** When multiplying fractions, always multiply the divisor by the multiplicative inverse of the dividend.
6. **T** F When multiplying rational expressions, the numerator of the product is the product of the numerators.
7. T **F** If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions, then $\frac{a}{b} + \frac{c}{d} = \frac{a+c}{bd}$.
8. **T** F To factor a polynomial means to write it as a product.
9. **T** F Subtraction of rational expressions is defined as an addition.
10. T **F** $(x+4)^2 = x^2 + 16$.
11. The set of **rational** Numbers consists of all numbers which can be written as fractions.
12. A **rational** expression is an expression that can be written as the quotient of two polynomials with the denominator not zero.
13. If $\frac{a}{b}$ and $\frac{c}{d}$ are fractions, then their quotient is defined by $\frac{a}{b} \div \frac{c}{d} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right)$
14. If $\frac{a}{b}$ and $\frac{c}{b}$ are fractions with the same denominator, then their sum is defined by $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$
15. Squaring a binomial will yield a trinomial whose first and last terms are **squares**.
16. The **hypotenuse** of a right triangle is the side opposite the right angle.
17. If a and b are real numbers and $ab = 0$, then **a = 0** or **b = 0**.
18. The **Greatest Common Factor** of two or more monomials is the product of all numbers and letters which divide each of the monomials.
19. If a, b and d are Real numbers and neither b nor d is 0, then $\frac{a}{b} = \frac{\boxed{ad}}{bd}$.
20. The **opposite** of $x - y$ is $y - x$.

USE = SYMBOLS WHERE APPROPRIATE

21. Factor Completely. $x(y - 2) + (y - 2) = (y - 2)(x + 1)$

Carefully observe the use of the = symbol. Just writing $(y - 2)(x + 1)$ somewhere on the page is just wrong!

22. Factor Completely. $x^3 + 5x^2 - 9x - 45 = x^2(x + 5) - 9(x + 5) = (x + 5)(x^2 - 9) = (x + 5)(x + 3)(x - 3)$

Carefully observe the use of the = symbol. Whether you present your work horizontally or vertically, the = symbols are a **requirement**.

23. Factor Completely. $x^2 + 4x + 5$ This quadratic is prime.

Carefully observe the use of WORDS. Words are useful for communication!

24. Factor Completely. $64x^2 - 100 = 4(16x^2 - 25) = 4(4x + 5)(4x - 5)$

Carefully observe the use of the = symbol.

25. Factor Completely. $16x^2 + 8x + 1 = (4x + 1)^2$

Carefully observe the use of the = symbol.

26. Factor Completely. $(x + 3)^2 - 4^2 = (x + 3 + 4)(x + 3 - 4) = (x + 7)(x - 1)$

Carefully observe the use of the = symbol.

Observe the factoring was accomplished by recognizing the original expression as the difference of two squares.

27. Solve $x^2 - 9x - 36 = 0$

$(x - 12)(x + 3) = 0$ ←to omit =0 here is incorrect

By the Zero Factor Property ←to omit this observation is not acceptable

$x - 12 = 0$ OR $x + 3 = 0$

$x = 12$ OR $x = -3$

The solution set for the equation is {12, -3} ←to state that $x = \{12, -3\}$ is wrong. x represents a number not a set. It is correct to write $x \in \{12, -3\}$.

28. Solve $12x^2 - 17x + 6 = 0$

$(4x - 3)(3x - 2) = 0$ ←to omit $=0$ here is incorrect

By the Zero Factor Property ←to omit this observation is not acceptable

$4x - 3 = 0$ OR $3x - 2 = 0$

$x = \frac{3}{4}$ OR $x = \frac{2}{3}$

The solution set for the equation is $\left\{\frac{3}{4}, \frac{2}{3}\right\}$ ←to state that $x = \left\{\frac{3}{4}, \frac{2}{3}\right\}$ is wrong. x represents a number not a set. It is correct to write $x \in \left\{\frac{3}{4}, \frac{2}{3}\right\}$.

29. Simplify $\frac{x^2 + 6x - 40}{x + 10} = \frac{(x - 4)(x + 10)}{x + 10} = x - 4$ if $x \neq -10$

30. Multiply and Simplify

$$\begin{aligned} \left(\frac{9x + 9}{4x + 8}\right)\left(\frac{2x + 4}{3x^2 - 3}\right) &= \frac{9(x + 1)(2)(x + 2)}{4(x + 2)(3)(x^2 - 1)} = \frac{9(x + 1)(2)(x + 2)}{4(x + 2)(3)(x - 1)(x + 1)} = \left(\frac{9}{3}\right)\left(\frac{2}{4}\right)\left(\frac{x + 2}{x + 2}\right)\left(\frac{x + 1}{x + 1}\right)\left(\frac{1}{x - 1}\right) \\ &= (3)\left(\frac{1}{2}\right)(1)(1)\left(\frac{1}{x - 1}\right) = \frac{3}{2(x - 1)} \end{aligned}$$

The steps in blue are not absolutely necessary, but serve to insure that the cancellations are done properly. The material shown in red (including the = symbols) are absolutely necessary for proper communication. For this kind of exercise, it is NEVER a wise move to multiply first. Look for factorizations and cancellations before multiplication.

31. Change to Multiplication (don't do anything else!)

$$\begin{aligned} \frac{x^2 - 4}{3x + 6} \div \frac{2x^2 - 8x + 8}{x^2 + 4x + 4} \\ \downarrow \\ \frac{x^2 - 4}{3x + 6} \bullet \frac{x^2 + 4x + 4}{2x^2 - 8x + 8} \end{aligned}$$

$$32. \text{ Add and Simplify } \frac{7}{2xy^2} + \frac{1}{2xy^2} = \frac{7+1}{2xy^2} = \frac{8}{2xy^2} = \frac{4}{xy^2}$$

The blue step is for instructional purposes and should no longer be necessary.

33. Add and Simplify

$$\begin{aligned} \frac{x+1}{x^2-6x+8} + \frac{3}{x^2-16} &= \frac{x+1}{(x-2)(x-4)} + \frac{3}{(x+4)(x-4)} \\ &= \frac{(x+1)(x-4)}{(x-2)(x+4)(x-4)} + \frac{3}{(x-2)(x+4)(x-4)} \\ &= \frac{(x+1)(x+4) + 3(x-2)}{(x-2)(x+4)(x-4)} = \frac{x^2 + 8x - 2}{(x-2)(x+4)(x-4)} \end{aligned}$$

Please observe the organized manner in which this addition is performed. Anything less is pure folly. Any other organization is probably inefficient. Presenting various calculations at random locations on the page is just plain wrong. Omitting the denominator for a couple of steps and then “slipping” it back into the discussion at the end is WRONG.

$$34. \text{ Simplify } \frac{\frac{5x}{x+2}}{\frac{10}{x-2}} = \left(\frac{5x}{x+2} \right) \left(\frac{x-2}{10} \right) = \frac{x(x-2)}{2(x+2)}$$

$$35. \text{ Simplify } \frac{x^{-1} + 2xy^{-1}}{x^{-2} - x^{-2}y^{-1}} \text{ Hint: Convert to an improper fraction and then simplify.}$$

$$\frac{x^{-1} + 2xy^{-1}}{x^{-2} - x^{-2}y^{-1}} = \frac{\frac{1}{x} + \frac{2x}{y}}{\frac{1}{x^2} - \frac{1}{x^2y}} = \frac{\frac{y + 2x^2}{xy}}{\frac{y-1}{x^2y}} = \left(\frac{y + 2x^2}{\cancel{xy}} \right) \left(\frac{\cancel{x^2y}}{y-1} \right) = \frac{x(y + 2x^2)}{y-1}$$