Function Notation Exercises Solutions

Part I:

a) Write each of the following mathematical statements using normal English prose.
b) Write what each of these statements tells you about the graph of the function.

Example: The statement f(3) = 7 should be written as:

a) The unique range element associated with the domain element 3 by the function f is 7.
b) The point (3, 7) is on the graph of f.

1) f(2) = 8

a) The unique range element associated with 2 by the function f is equal to 8.

b) The point (2, 8) is on the graph of the function f.

2) f(5) = (5)(67)

a) The unique range element associated with 5 by the function f is equal to 5 times 67.

b) The point (5, (5)(67)) is on the graph of the function f. Note that this point may also be specified as (5, 335)

3) $f(\pi) = \sqrt{23}$

a) The unique range element associated with π by the function f is equal to $\sqrt{23}$.

b) The point $(\pi, \sqrt{23})$ is on the graph of the function f. Note that the second coordinate of this point is an irrational number between 4 and 5 and is closer to 5 than to 4.

4) f(x) = 3x + 1

a) The unique range element associated with x by the function f is equal to 3x + 1. Note that this is in fact a general rule for the function f. That rule can be stated as follows: The unique range element associated with a domain element by the function f is one plus three times the domain element.

b) The point (x, 3x + 1) is on the graph of the function f.

5) $f(x) = 2^x + 5^x$

a) The unique range element associated with x by the function f is equal to 2 raised to the x^{th} power plus 5 raised to the x^{th} power. Note that this is in fact a general rule for the function f. Later in this course it will become clear that this is an example of an exponential function.

b) The point $(x, 2^x + 5^x)$ is on the graph of the function f. The graph of f consists of all such points.

6) $f(x) = \pi^x$

a) The unique range element associated with x by the function f is equal to π raised to the x^{th} power. Note that this is in fact a general rule for the function f. Later in this course it will become clear that this is an example of an exponential function.

b) The point (x, π^x) is on the graph of the function f. The graph of f consists of all such points.

7) $f(x) = x^{\pi}$

a) The unique range element associated with x by the function f is equal to x raised to the π^{th} power. Note that this is in fact a general rule for the function f. This is not an exponential function because the exponent is not variable.

b) The point (x, x^{π}) is on the graph of the function f. The graph of f consists of all such points.

8) $f \circ g(x) = f(g(x))$

a) The unique range element associated with x by the function $f \circ g$ is equal to the unique range element associated with the domain element g(x) by the function f. Note however, that although g(x) is a domain element for the function f, with respect to the function g it is the unique range element associated with the domain element x by the function g. It is almost impossible to write this in a meaningful way with ordinary English, but here is an attempt.

The unique range element associated with x by the function $f \circ g$ is equal to the unique range element associated, by f, with the unique range element associated with the element x of the domain of g by the function g. This references the standard diagram for composition of function (later in this course).

b) Has no graphical interpretation.

9) $f \circ g(2) = 5$

a) The unique range element associated with 2 by the function $f \circ g$ is equal to 5.

b) The point (2, 5) is on the graph of the function $f \circ g$.

10) f(7) > 48

a) The unique range element associated with 7 by the function f is greater than 48.

b) The point (7, f(7)) is on the graph of the function f and is above the horizontal line y = 48.

11) $f(2) < 5^2$

a) The unique range element associated with 2 by the function f is greater than 25.

b) The point (2, f(2)) is on the graph of the function f and is above the horizontal line y = 25.

12) f(3) > g(x)

a) The unique range element associated with 3 by the function f is greater than unique range elements associated with domain elements by the function g.

b) The point (3, f(3)) is on the graph of the function f and is above every point on the graph of g.

Part II:

a) Write each of the following mathematical statements using correct mathematics notation.

Example: The statement; "The range element associated with the domain element 4 by the function f is 7" should be written as: f(4) = 7.

1) The range element associated with the domain element 8 by the function f is 11.

This statement can be written as f(8) = 11.

2) 45 is the range element associated with the domain element 9 by the function g.

This statement can be written as g(9) = 45.

3) The rule for the function f is: The range element associated with a domain element is the domain element raised to the 11^{th} power.

The rule for the function f is: $f(x) = x^{11}$

4) The range element associated with the domain element 14 is less than the range element associated with the domain element $\frac{3}{\sqrt{7}}$ by the same function f.

This statement can be written as $f(14) < f\left(\frac{3}{\sqrt{7}}\right)$.

5) Two functions f and g have the same domains and for each domain element, the range element associated with it by the function g is greater than the range element associated with it by the function f.

Functions f and g have the same domains and for every element of this common domain g(x) > f(x).

6) The point (3, 5) is on the graph of the function h.

This statement can be written as f(3) = 5.

7) The point $\left(\frac{3}{4}, \frac{2\pi}{3}\right)$ is on the graph of the function jim.

This statement can be written as $\text{jim}\left(\frac{3}{4}\right) = \frac{2\pi}{3}$

8) 86 is a zero of the function k.

This statement can be written as k(86) = 0.

9) The function f is a linear function.

The statement may be written as: The rule for the function f has the form f(x) = mx + b where m and b are real numbers.

10) The function f is a quadratic function.

The statement may be written as: The rule for the function f has the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$.