

NAME: _____ Score _____ /100
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SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct.

2 pts. each for 1 – 25. 5 pts. each for all others.

1. T **F** A solution of the equation $x^3 - x^2 - x - 3 = 0$ is 2.
2. **T** F $[2, 4) = \{x | x \in \mathbb{R} \text{ and } 2 \leq x < 4\}$
3. T **F** The graph of a linear equation in one variable is a line.
4. T **F** The complex component of $7 - 8i$ is $-8i$.
5. **T** F The sum of a complex number and its conjugate is a real number.
6. **T** F $3 + 9 = 17$ is a mathematical equation.

Circle the symbol for the smallest set of numbers which contains the number given at the left.

The Symbols are standard: **C** is the complex numbers, **R** is the real numbers, **F** is the irrational numbers, **Q** is the rational numbers, **Z** is the integers, **W** is the whole numbers, and **N** is the natural numbers, and **P** is the prime numbers.

7. The smallest set which contains 3 is **C R F Q Z W N P**
8. The smallest set which contains $\frac{2}{5} - 7i$ is **C R F Q Z W N P**
9. The smallest set which contains $-\frac{7}{8}$ is **C R F Q Z W N P**
10. The smallest set which contains 6 is **C R F Q Z W N P**
11. The smallest set which contains $\sqrt{7}$ is **C R F Q Z W N P**
12. The smallest set which contains -43 is **C R F Q Z W N P**

Fill in each of the blanks to make the statements true.

13. The exact area of a circle with radius 5 is 25π .
14. Write the formula for the volume of a cylinder. $V = \pi r^2 h$. (Your answer must be a formula.)
15. A linear equation in one variable is an equation which may be written in the form $ax + b = 0$.
16. A quadratic equation in one variable is an equation which may be written in the form $ax^2 + bx + c = 0$.
17. A complex number is a number that can be written in the form $a + bi$ where a and b are real numbers and $i = \sqrt{-1}$.

18. A number (or numbers) that makes an equation **true** when substituted for the variable (or variables) is called a **solution** of the equation.
19. If any expression is added to both sides of an equation the resulting equation is **equivalent** to the original equation.
20. If both sides of an equation are squared the solution set of the resulting equation **contains** the solution set of the original equation.
21. Use set builder notation to describe the set of rational numbers less than 8 and greater than 5.
 $\{x \mid x \in \mathbb{Q} \text{ and } 5 < x < 8\}$

Circle all the words which could be used to correctly complete the sentence.

22. $2x - 7 = 0$ is a (**linear** quadratic identity **conditional** contradiction) equation.
23. $(x + 2)(x - 3) = x^2 - x - 6$ is a (linear **quadratic identity** conditional contradiction) equation.
24. $x^2 - x - 6 = 0$ is a (linear **quadratic** identity **conditional** contradiction) equation.
25. $3x + 4 = 3x + 2$ is a (**linear** quadratic identity conditional **contradiction**) equation.
26. Write the quadratic formula.

Solutions of the quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

27. Write the multiplicative inverse of the complex number $5 - 2i$.

The multiplicative inverse of $5 - 2i$ is $\frac{5 + 2i}{5^2 + 2^2} = \frac{5 + 2i}{29}$

Write the zero factor property.

If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

28. Sketch the graph of $3x - 6 = 0$



29. Compute the product $(3 + 2i)(1 - 5i)$. Your answer should be written in standard form. (**Show your work. no work: no credit**)
 $(3 + 2i)(1 - 5i) = 3 - 15i + 2i - 10i^2 = 3 + 10 - 13i = 13 - 13i$

30. Change $(3 + 2i) \div (1 - 5i)$ into an equivalent multiplication problem. (**DO NOT perform the multiplication.**)

$$\begin{aligned} &(3 + 2i) \div (1 - 5i) \\ &\quad \downarrow \quad \downarrow \\ &(3 + 2i) \cdot \left(\frac{1 + 5i}{1^2 + 5^2} \right) \end{aligned}$$

31. Use the Quadratic formula to solve the equation $x^2 + x - 1 = 0$. (Show your work. no work: no credit)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$

32. Solve the equation $2x^2 - 13x + 15 = 0$. (Do not use the quadratic formula or completing the square.) (Show your work. no work: no credit)

$$2x^2 - 13x + 15 = 0$$

$$(2x - 3)(x - 5) = 0$$

By the Zero Factor Property

$$2x - 3 = 0 \text{ OR } x - 5 = 0$$

$$x = \frac{3}{2} \text{ OR } x = 5$$

The solutions are $\frac{3}{2}$ and 5.

The solution set is $\left\{\frac{3}{2}, 5\right\}$

33. A cone has radius 3 feet and volume 36π cubic feet. What is the height? (Show your work. no work: no credit)

$$V = \frac{1}{3}\pi r^2 h \text{ so in this problem } 36\pi = \frac{1}{3}\pi 3^2 h \text{ which is easily solved for } h \text{ to obtain } h = \frac{36\pi}{3\pi} = 12$$

The cone is 12 feet high.

38. How much water must be added to 20 ounces of a 15% acid solution to reduce it to a 10% acid solution? Fill in the blanks to provide a correct and complete analysis of this question.

Solution: Let x be the amount of water to be added.

The amount of the final solution is $x + 20$.

The amount of acid in the original solution is $(0.15)(20) = 3$.

The amount of acid in the final solution is 3.

The amount of acid in the final solution is $(0.1)(x + 20)$. Must be different than the previous blank.

We now have two expressions for the quantity.

The model is therefore $(0.1)(x + 20) = 3$.

Use ordinary means to solve this equation

$$(0.1)(x + 20) = 3$$

$$x + 20 = 3/(0.1) = 30$$

$$x = 10$$

Conclusion: 10 ounces of water must be added to obtain a 10% acid solution.