Meramec College Algebra TEST 2 Solution Fall 2008

NAME: _____Score_____/100
Please print

Circle T or F, whichever is correct. **2 pts. each for 1 – 25. 3 pts. each for all others unless otherwise noted.**

1. T **F** A zero of a function is in the range of the function.
2. T **F** Absolute value equations have only positive solutions.
3. T **F** The graph of a linear inequality in one variable is a point on the real number line.
4. **T** F The sum of two functions is a function.
5. T **F** The product of two functions is a range element.
6. T **F** The Law of Trichotomy is a property of equations.
7. **T** F A quadratic function is a polynomial function.

Fill in each of the blanks to make the statements true.

8. If f(3) = 8 and g(3) = -2 then (fg)(3) = -16.

9. The vertex of the graph of a quadratic function f is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

10. The graph of a linear function is a non-vertical line.

11. The graph of a quadratic function is a parabola which opens up or down.

12. The graph of a polynomial function is a continuous smooth curve with no sharp corners.

13. The solution set for $|x - 4| < 2$ is $\{x \mid 2 < x < 6\}$. **(use set builder notation)**

14. The solution set for $|x - 4| > 2$ is $(-\infty, 2) \cup (6, +\infty)$. **(use interval notation)**

15. The solution set for $|x - 4| = 2$ is $\{2, 6\}$. **(use the roster method)**

In questions 16 - 20 be as specific as possible.

16. The rule for a function is f(x) = 3x – 5. What kind of function is f? linear.

17. The rule for a function is $f(x) = \frac{4}{7}x^5 - 2x^3 + \sqrt{3}$. What kind of function is f? polynomial.

18. The rule for a function is $f(x) = 6x^2 - 5x + 2$. What kind of function is f? quadratic.

19. The graph of the identity function is a line.

20. The graph of the squaring function is a parabola.

Circle all the words which could be used to correctly complete the sentence.

21. f(x) = 2x – 7 is the rule for a (constant **linear** identity quadratic squaring cubing **polynomial**) function.

22. f(x) = x^2 – x – 6 is the rule for a (constant linear identity **quadratic** squaring cubing **polynomial**) function.

23. f(x) = x^5 – x^4 – 6 is the rule for a (constant linear identity quadratic squaring cubing **polynomial**) function.

24. f(x) = x^2 is the rule for a (constant linear identity **quadratic squaring** cubing **polynomial**) function.

25. f(x) = x is the rule for a (constant **linear identity** quadratic squaring cubing **polynomial**) function.

26. If the rules for two functions f and g are f(x) = 4x and g(x) = x^2 – x + 1, then the rule for the product fg is fg(x) = **$4x^3 – 4x^2 + 4x$**

27. Write the compound inequality which does not involve absolute value and is equivalent to |2x – 3| < 8 . **Do NOT solve.**

–8 < 2x – 3 < 8

28. Complete the statement of the Zero Factor Property.

If a and b are real numbers and ab = 0, then a = 0 or b = 0.

For Problems 29 – 35 you must show your work or state reasons for your conclusions.

29. Find the rule for the linear function whose graph has slope 3 and passes through (3, –5)

Use $y – y_1 = m(x – x_1)$	Because the function is linear, its rule has the form f(x) = mx + b
y + 5 = 3(x – 3)	Because m = 3, we have f(x) = 3x + b
y = 3x – 14	Because (3, –5) is on the graph, f(3) = –5 but f(3) = 3(3) +b
The rule for the function is	So 9 + b = –5 from which we obtain b = –14
f(x) = 3x – 14	Therefore the rule is f(x) = 3x – 14

30. What is the domain of the function whose rule is $f(x) = \frac{3x + 6}{x – 5}$

The domain is all real numbers for which the rule makes sense. Therefore the domain is all real numbers except those that create a 0 in the denominator.

The domain is all real numbers except 5.

The domain is $\{x \in R \mid x \neq 5\}$

31. What are the zeros of the function whose rule is $f(x) = |3x – 5|$

To find the zeros of f we must solve the equation resulting from f(x) = 0.

Therefore we must solve the equation $|3x – 5| = 0$ which is true if and only if 3x – 5 = 0 or $x = \frac{5}{3}$.

The zero of f is $\frac{5}{3}$

32. Calculate the exact length of the line segment joining (4, –3) and (–2, 5).

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(4+2)^2 + (-3-5)^2} = \sqrt{36+64} = \sqrt{100} = 10$

33. Use completing the square to determine the center and radius of the circle whose equation is

$x^2 + y^2 + 2x - 6y + 7 = 0$

a) The center is (-1, 3) b) The radius is $\sqrt{3}$

Show your work here.

$x^2 + y^2 + 2x - 6y + 7 = 0$

$x^2 + 2x + y^2 - 6y = -7$

$x^2 + 2x + 1 + y^2 - 6y + 9 = -7 + 1 + 9$

$(x+1)^2 + (y-3)^2 = 3$

34. The rule for the function f is $f(x) = x^3 - 2x^2 + x - 5$ and the point (2, k) is on the graph of f. What is the value of k?

Because the point (2, k) is on the graph of f, $k = f(2) = 2^3 - 2(2^2) + 2 - 5 = -3$

35. The rule for the function f is $f(x) = (x + 2)(x - 2i)(x + 2i)$. What are the x-intercepts of the graph of f.

The x-intercepts of the graph of f are at the real zeros of f. The only real zero of f is –2.
Therefore the x-intercept of the graph of f is (–2, 0)

36. Consider the graph of a function f shown in Figure 1.

a. Where is $f(x) < 0$? **Use interval notation.**
   $f(x) < 0$ if $x \in (-2,3) \cup (4, +\infty)$
b. Where is $f(x) = 0$? **Use the roster method.**
   $f(x) = 0$ if $x \in \{-2, 3, 4\}$
c. Where is $f(x) > 0$? **Use interval notation.**
   $f(x) < 0$ if $x \in (-\infty, -2) \cup (3, 4)$

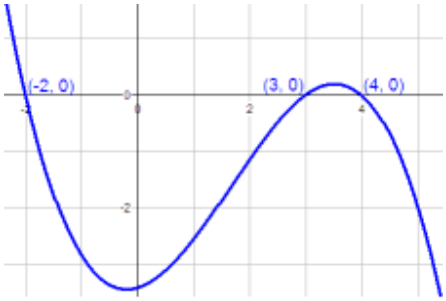

Figure 1

37. (5 points)The rule for a function f is $f(x) = -3x^5 - 22x^4 + 6x^3 - 7x^2 + 8x - 5$. Complete the following statements about f.

a. The graph of f "tries" to cross the x-axis five times.

b. The graph of f can cross the x-axis no more than five times.

c. As $x \longrightarrow +\infty, f(x) \longrightarrow -\infty$

As $x \longrightarrow -\infty, f(x) \longrightarrow +\infty$

d. The graph of f must cross the x-axis at least one times.

38.(6 pts)Definition: A function consists of three things;

- A set called the domain
- A set called the range
- A rule which associates each element of the domain with a unique element of the range.

39.