Meramec College Algebra TEST 1 Solution Spring 2008

NAME: ______________________________ Score______________________/100

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SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct. **1 pts. each for 1 – 25. 5 pts. each for 26 – 37. 38 is 13 pts. 39 is 2 pts.**

1. T F The opposite of $-\frac{3}{4}$ is $-\frac{4}{3}$.

2. T F $\mathbf{N} \subset \mathbf{R}$.

3. T F $\{x | x \in R \text{ and } 1 < x < 5\} = \{2, 3, 4\}$

4. T F The graph of 3x + 2 = 0 is a line.

5. T F $\{3, b, x, 5\}$ is an example of the roster method for specifying a set.

6. T F If both sides of an equation are multiplied by x – 2, the resulting equation is equivalent to the original equation.

7. T F Every real number is a rational number.

8. T F Division by 0 is undefined.

9. T F If x is a real number, then $-x$ is negative.

10. T F If Q is the set of rational numbers and F is the set of irrational numbers, then $Q \cup F = R$.

Fill in each of the blanks to make the statements true.

11. A set is a collection of objects.

12. The null set is the set with no elements.

13. The set A is a subset of the set B if every element of set A is an element of set B.

14. The process to solve a linear equation in one variable is to generate a sequence of equations each equivalent to the previous equation until a simplest equation is obtained.

15. A number (or numbers) that makes an equation true when substituted for the variable (or variables) is called a solution of the equation.

16. A linear equation in one variable is an equation that can be written in the form ax + b = 0 where a and b are real numbers with a not zero.

17. The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

18. A real number which is not rational is irrational.

19. A prime number is a natural number greater than 1 which has only 1 and itself as factors.

20. A quadratic equation in one variable x is an equation which may be written in the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and a is not zero

21. The Pythagorean Theorem states that if a and b are the lengths of the legs of a right triangle with hypotenuse of length c, then $a^2 + b^2 = c^2$.

22. If a and b are real numbers and ab = 0, then a = 0 or b = 0.

23. Use the roster method to describe $\{x \mid x \in Z \text{ and } |x| < 3\}$ {–2, –1, 0, 1, 2}.

24. What is the solution set for |3x – 7| = –5 ? the null set (HINT: Think)

25. Insert the correct symbol (<, =, or >) in the blank.
If x and y are real numbers and x < y, then –3x > –3y

26. 42 is 14% of what number?
Use A = PB to obtain 42 = .14B.
Solve for B to obtain $B = \frac{42}{.14} = \frac{4200}{14} = 300$

27. List all of the possible subsets of { 1, 2, 3 }---There are eight subsets including the set { 1, 2, 3 }itself and the empty set. So you need to list six other subsets. Use the roster method of specifying the sets.

a. {1} b. {2} c. {3} d. {1,2} e. {1,3} f. {2,3}

28. Find the exact area of the circle with center (–4, 5) and radius 3
Use $A = \pi r^2$ to obtain $A = \pi 3^2 = 9\pi$

29. Show that 1 is a solution of the equation $x^5 + 3x^4 - 2x^3 + 6x^2 - 12x + 4 = 0$.
Replace x with 1 to obtain 1 + 3 – 2 + 6 – 12 + 4 = 0 which is a TRUE statement.
Therefore 1 is a solution of the equation.

30. A can manufacturer has a contract to make cylindrical cans with a radius of 4 inches and a volume of 48π cubic inches. What should be the height of the cans?

Use $V = \pi r^2 h$ to obtain $48\pi = 16\pi h$ which may be solved for h to obtain $h = \frac{48\pi}{16\pi} = 3$

31. A particular equation in one variable x has { -2, 1, 4} as its solution set. Sketch its graph.

32. Find the solution set for $2x^2 + x - 1 = 0$. Hint: Factoring works.

$2x^2 + x - 1 = 0$
$(2x - 1)(x + 1) = 0$
By the Zero Factor Property
$2x - 1 = 0$ OR $x + 1 = 0$
$2x = 1$ OR $x = -1$
$x = \frac{1}{2}$ OR $x = -1$
The solution set is $\left\{\frac{1}{2}, -1\right\}$

33. Solve the equation $\boldsymbol{K} = \frac{6}{7}\left(\boldsymbol{R} - \frac{7}{9}\right)$ for R.

Multiply both sides by $\frac{7}{6}$ to obtain $\frac{7}{6}K = R - \frac{7}{9}$

Add $\frac{7}{9}$ to both sides to obtain $\frac{7}{6}K + \frac{7}{9} = R$

34. Solve the equation $(x - 1)(x + 3)(x - 4) = 0$.

By the Zero Factor Property
$x - 1 = 0$ OR $x + 3 = 0$ OR $x - 4 = 0$
$x = 1$ OR $x = -3$ OR $x = 4$
The solution set is { 1, -3, 4}

35. Use the Quadratic formula to solve the equation $x^2 + x - 1 = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - (4)(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$

36. A rectangular garden is 25 ft. wide. If its area is 1125 sq. ft., what is its length?

Use A = xy to obtain 1125 = 25y which can be solved for y to obtain $y = \frac{1125}{25} = 45$

37. Solve the equation $3x - \frac{5}{7} = \frac{2}{3}x + \sqrt{3}$ (I want an exact solution—not a decimal approximation.)

Add $-\frac{2}{3}x$ to both sides to obtain $3x - \frac{5}{7} - \frac{2}{3}x = \sqrt{3}$

Then add $\frac{5}{7}$ to both sides to obtain $3x - \frac{2}{3}x = \sqrt{3} + \frac{5}{7}$

Which can be rewritten as $\left(3 - \frac{2}{3}\right)x = \frac{7\sqrt{3}}{7} + \frac{5}{7}$

Performing the additions then yields $\frac{7}{3}x = \frac{5 + 7\sqrt{3}}{7}$

Now multiply both sides by $\frac{3}{7}$ to obtain the solution $x = \left(\frac{3}{7}\right)\left(\frac{5 + 7\sqrt{3}}{7}\right) = \frac{15 + 21\sqrt{3}}{49}$

38. On this question I will lead you through a proper way of writing a solution to a question. You are to supply the details by filling in each blank. **No computations are required or even desired.**
Problem: What quantity of a 60% acid solution must be mixed with a 30% acid solution to produce 300 mL of a 50% acid solution ?

(*) and (**) should not be the same.

Analysis:
Let x be the amount (measured in milliliters) of 60% to be added.
The volume of the final mixture will be 300 mL.
The amount of acid in the final solution is (0.5)(300). (*)
The amount of acid contributed by the 60% solution is (0.6)x.
The amount of the 30% solution will be 300 – x mL.
The amount of acid contributed by the 30% solution is (0.3)(300 – x).
The amount of acid in the final solution is (0.6)x + (0.3)(300 – x) = 300. (**)
We now have the amount of acid in the final solution written in two ways.

Therefore the mathematical model for this concentration problem is the linear equation in one variable
(Insert the Model/equation here) (0.6)x + (0.3)(300 – x) = 300

Solution: (I have solved the equation so you don't need to)
Ordinary methods now may be used to solve this equation to obtain x = 200.

Conclusion: (Must be based on the correct solution as stated above.)
200 milliliters of 60% solution must be added to 100 milliliters of 30% solution to obtain 300 milliliters of 50% solution.

39. A grocer mixes peanuts that cost \$2.49 per pound and walnuts that cost \$3.89 per pound to make 100 pounds of a mixture that costs \$3.19 per pound. How much of each kind of nut is put into the mixture?
Solution:
Let x be the amount of peanuts to be put into the mixture.
Then 100 – x is the amount of walnuts put into the mixture.
The cost of the peanuts in the mixture is 2.49x
The cost of the walnuts in the mixture is 3.89(100 – x)
The total cost contributed by the peanuts and walnuts is 2.49x + 3.89(100 – x).
However, the total cost of the final mixture is required to be (3.19)(100) or 319.
This completes the analysis of the problem and we are now in a position to model the mixture problem with an equation
Write that equation. Do not solve the equation.
2.49x + 3.89(100 – x) = 319