Meramec College Algebra TEST 2 Solution Spring 2008

NAME: ________________________________ Score____________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct. 3 pts. each for 1 – 12. 7 pts. each for 13 – 21.

1. T **F** Every quadratic equation in one variable has two solutions.

2. T **F** The complex component of 8 – 2i is – 2i.

3. **T** F The norm of a complex number is a real number.

4. T **F** If both sides of an equation are squared, the resulting equation has the same solution set as the original equation.

5. T **F** The complex component of a complex number is a complex number.

6. **T** F Some quadratic equations have complex solutions.

Fill in each of the blanks to make the statements true.

7. A quadratic equation in one variable is an equation which may be written in the form $\mathbf{ax^2 + bx + c = 0}$ where a, b, and c are real numbers and a is not zero.

8. When both sides of an equation are squared the solution set of the resulting equation **contains** the solution set of the original equation.

9. If $b^2 - 4ac = 0$, the quadratic equation $ax^2 + bx + c = 0$ has **one** solution.

10. State the Quadratic Formula.

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

11. The norm of –5 –3i is $\mathbf{(-5)^2 + (-3)^2}$.

12. What is the solution set for |3x – 7| = –5 ? **The null set. Because absolute value cannot be negative.**

13. Compute the sum (3 – 2i) + (-7 + 5i) **= (3 – 7) + (-2 +5)i = -4 + 3i**

14. Compute the product (3 – 2i)(-7 + 5i) **= -21 +15i +14i -10i^2 = (-21 + 10) + (15 + 14)i = -11 +29i**

15. Compute the quotient(3 – 2i) ÷(-1 + 3i)

$$(3 - 2i) \div (-1 + 3i)$$

$$\downarrow \qquad \downarrow$$

$$(3-2i) \bullet \left(\frac{-1-3i}{(-1)^2+3^2}\right) = \frac{-3-9i+2i+6i^2}{10} = \frac{-9-7i}{10}$$

16. Solve the equation $x^2 + x - 12 = 0$ by factoring.

Solution:

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

By the Zero Factor Property

$x + 4 = 0$ OR $x - 3 = 0$

$x = -4$ OR $x = 3$

17. Solve the equation $x^2 - 4x + 2 = 0$ with the Quadratic Formula.

Solution: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)} = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$

18. Solve the equation $2x - 1 = \sqrt{2 - x}$.

Solution: $2x - 1 = \sqrt{2 - x}$ Square both sides

$4x^2 - 3x - 1 = 0$ Not equivalent to the original equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{(-3)^2 - 4(4)(-1)}}{2(4)} = \frac{3 \pm \sqrt{9 + 16}}{8} = \frac{3 \pm 5}{5} = 1 \text{ or } -\frac{1}{4}$$

Test 1: LHS: 2(1) - 1 = 1 RHS: $\sqrt{2 - 1} = \sqrt{1} = 1$ **LHS = RHS and therefore 1 is a solution of the equation**

Test $-\frac{1}{4}$**: LHS:** $2\left(-\frac{1}{4}\right) - 1 = -\frac{1}{2} - 1 = -\frac{3}{2}$**. Because this is negative, it cannot be equal to the RHS because the principle square root is positive. Therefore** $-\frac{1}{4}$ **is not a solution of the equation.**

The solution set for the original equation is {1}.

19. Solve the equation |2x – 5| = 8

Case 1: $2x - 5 \geq 0$

The the equation becomes

$2x - 5 = 8$

$2x = 13$

$x = \frac{13}{2}$

Case 2: $2x - 5 < 0$

The the equation becomes

$-(2x - 5) = 8$

$2x - 5 = -8$

$2x = -3$

$x = -\frac{3}{2}$

20. Solve the equation $x - 5\sqrt{x} + 6 = 0$

Solution: Let $k = \sqrt{x}$ then the equation becomes

$k^2 - 5k + 6 = 0$

(k – 2)(k – 3) = 0 and by the Zero Factor Property

k – 2 = 0 OR k – 3 = 0

k = 2 OR k = 3

If $k = 2$, then $2 = \sqrt{x}$ from which it follows that x = 4.

If $k = 3$, then $3 = \sqrt{x}$ from which it follows that x = 9

The solution set is {4, 9}

21. Solve the equation $\frac{1}{x-1} - \frac{2}{x^2} = 0$

Solution:

$\frac{1}{x-1} - \frac{2}{x^2} = 0$ Multiply both sides by the LCD $x^2(x-1)$

$x^2 - 2(x-1) = 0$ May not be equivalent to the original equation

$x^2 - 2x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm \sqrt{4-8}}{2}$$

$$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$$

Since neither 1 + i nor 1 – i cause a denominator of zero, they both are solutions.