Meramec College Algebra TEST 2 Form B Solution Ssummer 2008

NAME: _____Score_____/100
Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

1 pt. each for 1 – 30. 5 pts. each for all others unless otherwise noted.

Circle T or F, whichever is correct.

1. T F Zeros of functions are not in the range of the function.
2. T F All zeros of functions are in the domain of the function.
3. T F If the point (3, 5) is on the graph of a function f, then f(3) = 5.
4. T F If f is the absolute value function, then its domain is all real numbers.
5. T F If f is a 6^{th} degree polynomial function, then its graph has at least one x-intercept.
6. T F If f and g are functions then fg is a function whose rule is fg(x) = f(x)g(x).
7. T F The cubing function has no inverse.
8. T F If a real zero of a polynomial function has odd multiplicity, the graph crosses the x-axis at that point.

Fill in each of the blanks to make the statements true.

9. A rational function is the quotient of two polynomial functions.

10. The graph of a polynomial function is a smooth continuous curve with no sharp corners.

11. If f and g are inverses of each other then

    a) $f \circ g(x) = f(g(x) = x$

    b) $g \circ f(x) = g(f(x) = x$

12. A function whose rule is $f(x) = \begin{cases} 3x & \text{if } x < 4 \\ x^2 & \text{if } x \geq 4 \end{cases}$ is called a piecewise defined function.

13. By convention if the domain is not explicitly stated, it is the largest set of the real numbers for which the rule makes sense.

14. The graph of a function f is the set of all points of the form (a, f(a)) where a is an element of the domain and f(a) is the corresponding range element.

15. If f and g are functions such that f(2) = 5, f(7) = 6, f(3) = 1, g(2) = 7, g(5) = 3, g(6) = 4 , then

    $f \circ g(5) = f(g(5)) = f(3) = 1$ (show your work on the line)

16. The function f whose rule is $f(x) = \dfrac{6x^4 - 5x^3 - 3x + 5}{2x^5 - 22x + 5}$ has a horizontal asymptote y = 0

17. The degree of the function f whose rule is $f(x) = 3x^3 - 7x^5 + 2x + 7$ is 5

Circle all the words which can be used to correctly complete the sentence. **Remember all polynomial functions are rational functions.**

18. f(x) = 7 is the rule for a (constant linear quadratic identity polynomial rational) function.

19. f(x) = x^2 + 2x + 1 is the rule for a (constant linear quadratic identity polynomial rational) function.

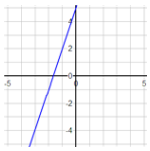
20. f(x) = x^5 + 5x + 6 is the rule for a (constant linear quadratic identity polynomial rational) function.

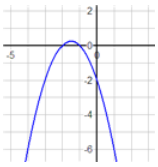
21. $f(x) = \frac{3x-4}{x^2+2x+1}$ is the rule for a (constant linear quadratic identity polynomial rational) function.

22. f(x) = x + 4 is the rule for a (constant linear quadratic identity polynomial rational) function.

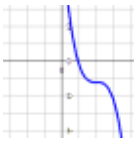
Match the following phrases with the graphs in problems 21 – 25 by writing the letter for the phrase in the blank.

(a) not a function (b) a function which does not have an inverse
(c) is a function which has an inverse

23.  This graph is the graph of c.

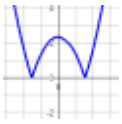
24.  This graph is the graph of b.

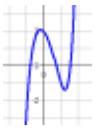
25.  This graph is the graph of a.

26.  This graph is the graph of c.

Match the following phrases with the graphs in problems 21 – 25 by writing the letter for the phrase in the blank.

(a) polynomial function (b) rational function
(c) quadratic function (d) piecewise defined function
(e) absolute value function (f) none of the above

27.  Is the graph of e.

28.  Is the graph of a.

29.  Is the graph of b.

30.  Is the graph of d.

31. What is the domain of the function whose rule is $f(x) = \frac{(x-3)(x+2)}{x(x-5)}$?

$D_f = \{x \in R \mid x \neq 0 \text{ and } x \neq 5\}$

32. What are the zeros of the function whose rule is $f(x) = \frac{x(x-3)(x-2)}{(x+5)}$?

The zeros are 0, 2, and 3.

33. What are the vertical asymptotes of the function whose rule is $f(x) = \frac{(x+3)(x-2)}{(x-5)(x+3)}$?

Remember asymptotes are lines not numbers.

The vertical asymptote is x = 5.

34. What is the horizontal asymptote (if there is one) of the function whose rule is $f(x) = \dfrac{2x^4 - 2x^3 + 10}{4x^4 - 5x^2 - 11x + 5}$

Remember asymptotes are lines not numbers.

The horizontal asymptote is $y = \frac{2}{4} = \frac{1}{2}$

35. What are the possible rational zeros of the function whose rule is $f(x) = 7x^3 + 2x^2 - 7x - 5$?

$p \in \{\pm 1, \pm 5\}$

$q \in \{\pm 1, \pm 7\}$

$\frac{p}{q} \in \left\{\pm 1, \pm \frac{1}{7}, \pm 5, \pm \frac{5}{7}\right\}$

36. The rule for a function f is $f(x) = -2x^7 - 22x^4 + 6x^3 - 7x^2 + 8x - 15$ .Complete the following statements about f.

a. The graph of f "tries" to cross the x-axis 7 times.

b. The graph of f can cross the x-axis no more than 7 times.

c. The graph of f must cross the x-axis at least 1 times.

d.

As $x \longrightarrow +\infty, f(x) \longrightarrow -\infty$

As $x \longrightarrow \infty, f(x) \longrightarrow +\infty$

37. Find the zeros of the function whose rule is $f(x) = x^2 + x - 1$. Show your work. no work: no credit

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$

38. Find the inverse of the function whose rule is $f(x) = 7x - 3$.

$f(x) = 7x - 3$

$y = 7x - 3$

$x = 7y - 3$

$x + 3 = 7y$

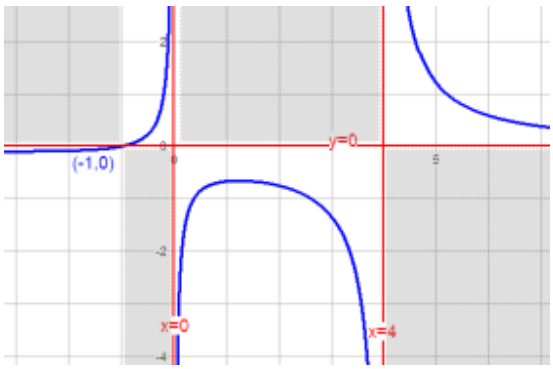
$y = \frac{x+3}{7}$

$f^{-1}(x) = \frac{x+3}{7}$

39. Find the rule for the linear function whose graph is the line through (–4, 3) with slope 7. Show your work. no work: no credit. Because the desired function is linear, it has the form f(x) = mx + b.
Because the slope is7, the desired function has the form $f(x) = 7x + b$. Call this (**)
Because (– 4, 3) is on the graph , f(– 4) = 3.
From (**) we compute $f(-4) = 7(-4) + b = -28 + b$.
Because we have two expressions for the same quantity f(–4), those two expressions must be equal.
$3 = -28 + b$ which implies $b = 31$.
The rule for the desired linear function is $f(x) = 7x + 31$.

40. An analysis of a function f reveals the following facts. **Shade excluded regions and Sketch the graph of f.**
Label important points.

a. f is a rational function
b. The real zero of f is -1
c. The vertical asymptotes are x = 0 and x = 4
d. The horizontal asymptote is y = 0
e. f(-2) < 0, $f(-\frac{1}{2}) > 0$, f(1) < 0, and f(5) > 0



41. An analysis of a function f reveals the following facts. **Sketch the graph of f.** **Label important points.**

a. f is a polynomial function of degree 5.
b. As $x \longrightarrow -\infty$, $f(x) \longrightarrow +\infty$
   As $x \longrightarrow +\infty$, $f(x) \longrightarrow -\infty$
c. The real zeros of f are -1, 0, and 3.
d. The multiplicity of -1 is 2.
e. The multiplicity of 3 is 2

42. Divide $x^3 - 7x^2 - x + 87$ by $x^2 - 10x + 29$. Show your work. no work: no credit.

$$
\begin{array}{r}
x + 3 \\
x^2 - 10x + 29 \overline{)\, x^3 - 7x^2 - x + 87} \\
\underline{x^3 - 10x^2 + 29x} \\
3x^2 - 30x + 87 \\
\underline{3x^2 - 30x + 87} \\
0
\end{array}
$$

43.(6 pts)Definition: A function consists of three things;

- A set called the domain
- A set called the range
- A rule which associates each element of the domain with a unique element of the range.

44. (4 pts)