Meramec College Algebra TEST 3 Solution Summer 2008

NAME: ________________________________ Score_____________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

1 pt. each for 1 – 20. 4 pts. each for all others unless otherwise noted.

Circle T or F, whichever is correct.

1. T F The functions ln and log are inverses.
2. T F ln(x + y) = ln(x) + ln(y).
3. T F det(AB) = det(A)det(B).
4. T F The rule for $\exp_5$ is $\exp_5(x) = x^5$.
5. T F Two systems of equations are equivalent if they have the same solution sets.
6. T F Every square matrix has an inverse.
7. T F The solution of a system of three equations in three variables is an ordered pair of numbers.
8. T F The graph of ln is entirely above the x-axis.
9. T F The point (0, 0) is in the solution set for the inequality 3x + 2y > -7.
10. T F $\ln \circ \exp(3x + 7) = 3x + 7$

Fill in each of the blanks to make the statements true.

11. A system S of equations consists of two linear equations in two variables. If the linear equations have different slopes, how many solutions does the system S have? One.

12. The domain of ln is the set of positive real numbers.

13. To find the point or points where the line x – y = 2 intersects the circle $x^2 + y^2 = 16$ it is necessary to solve

the system of equations $\begin{cases} x^2 + y^2 = 16 \\ x - y = 2 \end{cases}$

14. The rule for the exponential function base e is $\exp(x)=e^x$.

15. The zero of the ln function is 1.

16. The equation of the boundary line for the inequality 2x + 3y < 6 is 2x + 3y = 6

17. If the rule for a function f is $f(x) = 4^{x-6}$, then $f(8) = 4^{8-6} = 4^2 = 16$

Complete of the following statements of the elementary row operations for matrices.

18. Interchange two rows

19. Multiply a row by a non-zero constant and replace that row with the product.

20. Add a multiple of a row to another row and replace one but not both of the rows with that sum.

21. Write the coefficient matrix of the following system of equations: $\begin{cases} -2x+2y-4z=1 \\ 2x-5y-z=6 \\ 4x+2y-3z=5 \end{cases}$

$$\begin{bmatrix} -2 & 2 & -4 \\ 2 & -5 & -1 \\ 4 & 2 & -3 \end{bmatrix}$$

22. Use the substitution method to solve the system $\begin{cases} -x+y=2 \\ 4x-3y=-3 \end{cases}$

$$\begin{cases} -x+y=2 \\ 4x-3y=-3 \end{cases} \longrightarrow \begin{cases} y=x+2 \\ 4x-3y=-3 \end{cases} \longrightarrow \begin{cases} y=x+2 \\ 4x-3(x+2)=-3 \end{cases}$$

$$\longrightarrow \begin{cases} y=x+2 \\ x=3 \end{cases} \longrightarrow \begin{cases} y=3+2 \\ x=3 \end{cases} \longrightarrow \begin{cases} y=5 \\ x=3 \end{cases}$$

The solution for the system is the ordered pair (3,5).

23. Supply the missing entries by performing the indicated elementary row operation.

$$\begin{bmatrix} 3 & -1 & 1 & 0 \\ 5 & -3 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{5}R_2 \longrightarrow R_2} \begin{bmatrix} \boxed{3} & \boxed{-1} & \boxed{1} & \boxed{0} \\ \boxed{1} & \boxed{-\frac{3}{5}} & \boxed{0} & \boxed{\frac{1}{5}} \end{bmatrix}$$

23. Supply the missing entries by performing the indicated elementary row operation.

$$\begin{bmatrix} 2 & 1 & \frac{1}{5} & 0 \\ -4 & -3 & 0 & 1 \end{bmatrix} \xrightarrow{2R_1+R_2 \longrightarrow R_2} \begin{bmatrix} \boxed{2} & \boxed{1} & \boxed{\frac{1}{5}} & \boxed{0} \\ \boxed{0} & \boxed{-1} & \boxed{\frac{2}{5}} & \boxed{1} \end{bmatrix}$$

24. Consider the matrices. $A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $C = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$

Solve the matrix equation AX = C.

$$X = A^{-1}C = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 29 \\ -25 \\ 10 \end{bmatrix}$$

25. Compute the determinant of $\begin{bmatrix} 4 & 3 \\ 4 & 5 \end{bmatrix}$ $\det\left(\begin{bmatrix} 4 & 3 \\ 4 & 5 \end{bmatrix}\right) = (4)(5) - (4)(3) = 8$

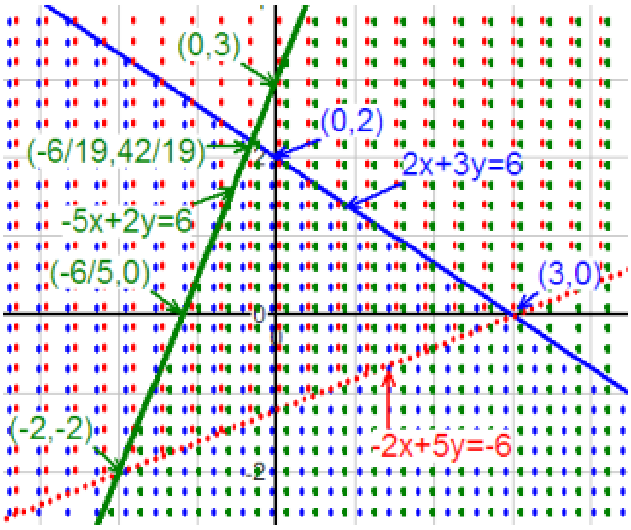
26. Write the 3×3 identity matrix. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

27. Perform the multiplication:

$$\begin{bmatrix} 1 & 2 & -4 \\ -2 & -3 & 3 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 12 & -8 \\ -16 & 7 \end{bmatrix}$$

28. **(10 points)** Sketch the graph of the solution for the system $\begin{cases} 2x + 3y \le 6 \\ -2x + 5y > -6 \\ -5x + 2y \le 6 \end{cases}$

Show and label the boundary lines, the important intercepts, points of intersection, and test points. Show all important computations.



29. **(10 points)** You do not need to do any computations. Simply fill in the blanks to describe the process for finding the inverse of a matrix.

To find the inverse of the matrix A = $\begin{bmatrix} 5 & 7 & 4 \\ 3 & -1 & 3 \\ 6 & 7 & 5 \end{bmatrix}$

Begin by adjoining the identity matrix to obtain the matrix $\begin{bmatrix} 5 & 7 & 4 & 1 & 0 & 0 \\ 3 & -1 & 3 & 0 & 1 & 0 \\ 6 & 7 & 5 & 0 & 0 & 1 \end{bmatrix}$ with order 3X6

The next step is to get a 1 in the11 position.

Then use that 1 to get 0 everywhere else in the first column

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 7/5 & 4/5 & 1/5 & 0 & 0 \\ 0 & -26/5 & 3/5 & -3/5 & 1 & 0 \\ 0 & -7/5 & 1/5 & -6/5 & 0 & 1 \end{bmatrix}$

The next step is to get a 1 in the 22 position.

Then use that 1 to get 0 everywhere else in the second column

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 0 & 25/26 & 1/26 & 7/26 & 0 \\ 0 & 1 & -3/26 & 3/26 & -5/26 & 0 \\ 0 & 0 & 1/26 & -27/26 & -7/26 & 1 \end{bmatrix}$

The next step is to get a 1 in the 33 position.

Then use that 1 to get 0 everywhere else in the third column

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 0 & 0 & 26 & 7 & -25 \\ 0 & 1 & 0 & -3 & -1 & 3 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{bmatrix}$

The inverse of A is matrix $A^{-1} = \begin{bmatrix} 26 & 7 & -25 \\ -3 & -1 & 3 \\ -27 & -7 & 26 \end{bmatrix}$

Which has order 3X3

30. Use the substitution method to solve the system $\begin{cases} x+2y+z=7 \\ -y+3z=9 \\ 2z=6 \end{cases}$

$$\begin{cases} \mathbf{x+2y+z=7} \\ \mathbf{-y+3z=9} \\ \mathbf{2z=6} \end{cases} \longrightarrow \begin{cases} \mathbf{x+2y+z=7} \\ \mathbf{-y+3z=9} \\ \mathbf{z=3} \end{cases} \longrightarrow \begin{cases} \mathbf{x+2y+3=7} \\ \mathbf{-y+9=9} \\ \mathbf{z=3} \end{cases} \longrightarrow \begin{cases} \mathbf{x+2y=4} \\ \mathbf{y=0} \\ \mathbf{z=3} \end{cases} \longrightarrow \begin{cases} \mathbf{x=4} \\ \mathbf{y=0} \\ \mathbf{z=3} \end{cases}$$

The solution for the system is the ordered triple (4, 0, 3)

31. The inverse of $A = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$ **is the matrix** $\mathbf{A^{-1}} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix}$

Use this information to solve the system $\begin{cases} -2x-3y+z=2 \\ -3x-3y+z=0 \\ -2x-4y+z=3 \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

The solution for the system is the ordered triple (2, -1, 3).

32. Solve the equation $\log_2(x) + \log_2(x + 1) = 1$

$\log_2(x) + \log_2(x + 1) = 1$

$\log_2[x(x + 1)] = 1$

$\log_2(x^2 + x) = 1$

$\exp_2 \circ \log_2(x^2 + x) = \exp_2(1)$

$x^2 + x = 2$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x + 2 = 0$ or $x - 1 = 0$

$x = -2$ or $x = 1$

The possible solutions are -2 and 1

If $x = -2$, then $\log_2(x) = \log_2(-2)$ is not defined because -2 is not in the domain of $\log_2$.

Therefore -2 is not a solution of the original equation.

On the other hand if $x = 1$, then both $\log_2(x)$ and $\log_2(x + 1)$ are defined.

33. Write $e^3 = y$ in logarithmic form.

exp(3) = y

ln(exp(3)) = ln(y)

3 = ln(y)

34. Solve the equation $e^x x^2 - e^x x - 2e^x = 0$.

$e^x x^2 - e^x x - 2e^x = 0$

$e^x(x^2 - x - 2) = 0$

$e^x(x - 2)(x + 1) = 0$

$e^x = 0$ or $x - 2 = 0$ or $x + 1 = 0$

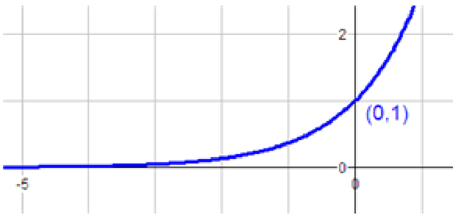
$e^x = 0$ or $x = 2$ or $x = -1$

$e^x = 0$ has no solution, so its solution set is the null set $\varnothing$, the solutions set for x = 2 is {2}, and the solution set for x = -1 is {-1}. The solution set for the original equation is therefore

$\varnothing \cup \{2\} \cup \{-1\} = \{2, -1\}$.

The solution set is {2, -1}

35. Sketch the graph of exp.



36. Sketch the graph of ln.