Meramec College Algebra TEST 1B Solution Fall 2009

NAME: ________________________________Score____________________________/100
Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION. NO WORK – NO CREDIT

Circle T or F, whichever is correct. **2 pts. each for 1 – 25. 4 pts. each for all others.**

1. T **F** Every rational number is a integer.

2. T **F** If an equation has a solution, then it is a conditional equation.

3. T **F** The graph of a linear equation in one variable is a line.

4. T **F** 2 is a solution of $x^3 + x^2 + x = 30$

5. T **F** If both sides of an equation are multiplied by 3x – 4, the resulting equation is equivalent to the original equation.

Circle the symbol for the smallest set of numbers which contains the number given at the left.

The Symbols are standard: **C** is the complex numbers, **R** is the real numbers, **F** is the irrational numbers, **Q** is the rational numbers, **Z** is the integers, **W** is the whole numbers, and **N** is the natural numbers.

6. The smallest set which contains -3 is **C R F Q Z W N** (answer: **Z**)

7. The smallest set which contains $\frac{2}{5} - 7i$ is **C R F Q Z W N** (answer: **C**)

8. The smallest set which contains $\sqrt{\frac{5}{2}}$ is **C R F Q Z W N** (answer: **F**)

9. The smallest set which contains 17 is **C R F Q Z W N** (answer: **N**)

10. The smallest set which contains $7i$ is **C R F Q Z W N** (answer: **C**)

11. The smallest set which contains $\frac{6}{5}$ is **C R F Q Z W N** (answer: **Q**)

Fill in each of the blanks to make the statements true.

12. The midpoint of the line segment joining the two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$\text{midpoint is } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

13. If the number 3 makes a particular equation true and 5 makes that same equation false, then that equation is a **conditional** equation.

14. Two equations are **equivalent** if they have the same solution sets.

15. The graph of an equation consists of all the points, and only those points, whose coordinates are **solutions** of the equation.

16. If any **expression** is added to both sides of an equation the resulting equation is equivalent to the original equation.

17. A linear equation in one variable is an equation that can be written in the form **ax + b = 0** where a and b are real numbers with a not zero.

18. The formula for the area of a trapezoid with bases b and B and height h is $A = \frac{1}{2}(B+b)h$.

19. The norm of a complex number a + bi is $\mathbf{a^2 + b^2}$.

20. The multiplicative inverse of a complex number a + bi is its **conjugate** divided by its **norm**.

Circle <u>all</u> the words which could be used to correctly complete the sentence.

21. 2x – 7 = 0 is a (**linear** quadratic identity **conditional** contradiction) equation.

22. 2x – 5 = 2x + 3 is a (**linear** quadratic identity conditional **contradiction**) equation.

23. $(x+2)(x+5) = x^2 + 7x + 10$ is a (linear **quadratic** **identity** conditional contradiction) equation.

24. $3x^2 + 4x = 3x + 2$ is a (linear **quadratic** identity c**onditional** contradiction) equation.

25. Complete the statement of the Distributive Property.
If a, b, and c are real numbers, then **a(b + c) = ab + ac**

26. Compute the sum (2 – 3i) + (6 + 4i). **Show the steps.**
(2 - 3i) + (6 + 4i) = (2 + 6) + (–3 + 4)i = 8 + i

27. Compute the product (2 – 5i)(4 – 2i). **Show the steps.**
$(2 - 5i)(4 - 2i) = (2)(4)\ (2)(-2)i + (-5)(4)i + (-5)(-2)i^2$
$8 - 4i - 20i + 10i^2 = 8 - 24i - 10 = -2 - 24i$

28. Change (2 + 4i) ÷ (3 – 2i) to the equivalent **multiplication (this is a correction)** problem.

(2 + 4i) ÷ (3 – 2i) = $(2+4i)\bullet\left(\frac{3+2i}{3^2+2^2}\right)=(2+4i)\bullet\left(\frac{3+2i}{13}\right)$

29. Solve 5x – 7 = 3x + 1. **Show the steps.**

5x – 7 = 3x + 1

2x – 7 = 1

2x = 8

x = 4

The solution set is {4}

30. Use the Quadratic formula to solve $3x^2 - 3x - 4 = 0$ **Show the steps.**

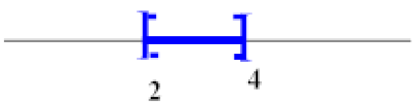
Use $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{3\pm\sqrt{(-3)^2-4(3)(-4)}}{2(3)}=\frac{3\pm\sqrt{9+48}}{6}=\frac{3\pm\sqrt{57}}{6}$$

31. Calculate the distance between the points (3, 5) and (7, –2).

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}=\sqrt{(7-3)^2+(-2-5)^2}=\sqrt{(4)^2+(-7)^2}=\sqrt{16+49}=\sqrt{65}$$

32. Sketch the graph of $\{x \mid 2\le x\le 4\}$



33. Consider the following process for solving $3x^2 - 7x = 6$:

Eq.1 $3x^2 - 7x = 6$

Eq.2 $3x^2 - 7x - 6 = 0$

Eq.3 $(3x + 2)(x - 3) = 0$

Eq.4 $3x + 2 = 0$ OR $x - 3 = 0$

Eq.5 $3x = -2$ OR $x - 3 = 0$

Eq.6 $x = -\frac{2}{3}$ OR $x = 3$

Eq.7 The solution set for the original equation is $\{3, -\frac{2}{3}\}$.

Now answer the following four questions about the above process.

33a. State the property which assures us that Eq.1 is equivalent to Eq.2 **Additive Property of Equations: If any expression is added to both sides of an equation the resulting equation is equivalent to the original equation.**

33b. State the property which assures us that Eq.3 is equivalent to the two equations joined with OR in Eq.4 **The Zero Factor Property: If a and b are real numbers and ab = 0, then a = 0 or b = 0.**

33c. State the property which assures us the first equation in Eq.5 is equivalent to the first equation in Eq.6 **Multiplicative Property of Equations: If both sides of an equation are multiplied by the same non-zero real number, the resulting equation is equivalent to the original equation.**

33d. The equations in Eq.6 are called **simplest** equations.

34. Solve the equation $\sqrt{2x - 1} = x - 2$

I will help you by providing some of the steps. You are to supply the others.

$\sqrt{2x - 1} = x - 2$ (I will square both sides)

$2x - 1 = x^2 - 4x + 4$

This second equation is not equivalent to the first equation. However, the solution set for the second

equation **contains the solution set for the original equation. (#)**

Now I will add -2x + 1 to both sides of the second equation to obtain

$x^2 - 6x + 5 = 0$ (You can take it from here. Finish solving the equation. Find the solution set.)

$(x - 5)(x - 1) = 0$

By The Zero Factor Property

$x - 5 = 0$ OR $x - 1 = 0$

$x = 5$ OR $x = 1$

These must be tested because of (#)

Test 5: $\sqrt{2(5) - 1} = 5 - 2$ is TRUE

Test 1: $\sqrt{2(1) - 1} = 1 - 2$ is FALSE

The solution set for the original equation is {5}

35. Find all values of x satisfying the following conditions.

$$y_1 = 2x^2 + 5x - 4 \qquad y_2 = -x^2 + 15x - 10 \quad \text{and} \quad y_1 - y_2 = 0$$

$y_1 - y_2 = 0$ implies $\left(2x^2 + 5x - 4\right) - \left(-x^2 + 15x - 10\right) = 0$

$3x^2 - 10x + 6 = 0$

Use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(3)(6)}}{2(3)} = \frac{10 \pm \sqrt{100 - 72}}{6} = \frac{10 \pm \sqrt{28}}{6} = \frac{10 \pm 2\sqrt{7}}{6} = \frac{5 \pm \sqrt{7}}{3}$$

36. Consider the formula $S = \dfrac{c}{1 - r}$. Solve for r.

$S = \dfrac{c}{1 - r}$

$S(1 - r) = c$

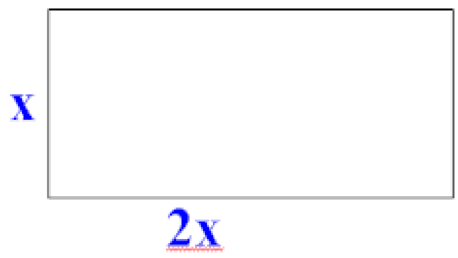
$S - Sr = c$

$-Sr = C - S$

$r = \dfrac{C - S}{-S} = \dfrac{S - C}{S}$

38. A field is twice as long as it is wide. The perimeter of the field is 300 feet. What are its dimensions?

Let x be the width
Then 2x is the length
Let P be the perimeter
Then P = 300
And P = 2x + 2(2x) =2x + 4x = 6x
We now have two expressions for the same quantity so they must be equal.
This yields 6x = 300
and x = 50
From which if follows that the field is 50ft. wide and 100 ft. long.



Comments About the Test

1.* We no longer use ● to indicate endpoints are included. Instead we use standard interval notation [ ]. Similarly we no longer use ○, we now use ( ) instead.

2. There is no chance of correctly using a formula if you don't know it.

3.* Language

- We add something to both sides.
- We multiply both sides by something.
- It is incorrect to write: add both sides by the same number.
- It is incorrect to write: multiply the same number to both sides.
- Use the word "solving correctly. eg. When finding the distance between points, you are computing not solving. eg. When multiplying two expressions, you are computing not solving.

4.* If two expressions are equal, you MUST indicate that fact with an equal symbol (=). If two expressions are not equal, you may not place and equal symbol between them.

5. Don't end a line with a symbol like =, <, or >.

6. * There are many quadratic equations but only one quadratic formula.

Definition: A quadratic equation in one variable is an equation which can be written in the form

$$ax^2 + bx + c = 0.$$

The quadratic formula is:

Quadratic Formula: The solutions of a quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Under no circumstances should you write $0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. This indicates a complete lack of understanding and will result in major point deduction.

7. The symbol ∅ means the empty set, consequently when writing mathematics it is improper and a potential source of misunderstanding if one draws a line through the symbol 0 for zero.

8. * When working a "word" problem:
At the very least there must be an English statement which identifies the variable you intend to use.
At the very least there must be a conclusion written in an English statement.
The work between these two statements must be organized so that it presents a coherent and logical argument leading from the hypothesis (the statement of the problem) to the conclusion. This part of the work frequently contains words and even entire English sentences.

9. * Mathematics is case sensitive. It is incorrect to change the case of a letter used in a problem. The symbol r is entirely different than R and it must be assumed that they represent different quantities.

10. Read and answer the question. Don't do all sorts of other things. For example if asked to change a division problem into the equivalent division problem, do that but then do not carry out the division.

11. * Never write two operation symbols next to each other. Don't write – – 2 when you mean – (– 2). Don't write – + 3 when you mean – (+3).

12. * The following is incorrect:

$x^2 + 6x + 5 = 0$
$(x + 1)(x + 5)$
By the Zero Factor Property
$x + 1 = 0$ OR $x + 5 = 0$

The 0 cannot just come and go in some willy-nilly fashion.

13. When adding or multiplying two expressions there is no solution set involved or solution for that matter. Look up the definition of solution and solution set.

14. * When adding or multiplying two complex numbers (or any other expressions) there is no excuse for setting that sum or product equal to zero. Think about it a bit. If asked to add 3 and 4 you would not compute 7 and then set that equal to zero to get 7 = 0 and decide there was no solution. If asked to compute $(3x + 2)(x – 5)$ you certainly would not set that product equal to zero and solve for x.

15. * It is impossible for an equation to be both linear and quadratic. Look at the definition of quadratic equation—why does it include the statement that the leading coefficient not be zero.

16. * The following is an example of an incorrect cancellation. $\frac{3+7x}{6} = \frac{1+7x}{2}$. To understand this just a little better look at $\frac{3+8}{6}$ which clearly is $\frac{11}{6}$ a number just a bit less than 2. Now look at what such creative cancellation produces. $\frac{3+8}{6} = \frac{\not{3}+8}{\not{6}} = \frac{1+8}{2} = \frac{1+\not{8}}{\not{2}} = \frac{1+4}{1} = 5$. Certainly 5 is not a bit less than 2. It must follow that this form of cancellation is incorrect. Cancellation is the process of identifying common factors OF (not in) the numerator and denominator, recognizing that their quotient is 1 and omitting the 1 from the product.

17. Algebra likes fractions and does not like decimals or mixed numbers. Avoid the use of decimals or mixed numbers when speaking Algebra.

18. * Guessing is not an algebraic method.

19. * Two equations CANNOT be equal.