

NAME: _____ Score _____/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION. NO WORK – NO CREDIT
Circle T or F, whichever is correct. (2 pts each)

1. **T** F A zero of a function is a domain element of the function.
2. **T** F A point is on the y-axis if and only if its first coordinate x is zero.
3. T **F** Some quadratic functions have vertices.
4. T **F** If $f(x) = 3x + 2$ and $g(x) = \frac{5}{x}$, then $f \circ g(x) = \frac{5}{3x + 2}$
5. **T** F Composition of functions is commutative.

In the following multiple choice questions, any number of choices may be correct. In each question at least one choice is correct. Circle ALL correct choices. (1 pt. for each choice.)

6. Consider the function whose rule is $f(x) = 3x - 7$.
 - a.** f is a linear function.
 - b.** The graph of f is a line.
 - c. The graph of f is a parabola.
 - d.** $f(2) = 8$.
 - e. f does not have an inverse.
 - f.** f has an inverse.
7. Consider the graph shown in Fig. 1.
 - a.** This is the graph of a function.
 - b. The graph passes the horizontal line test.
 - c. This is the graph of a quadratic function.
 - d. This is not the graph of a function.
 - e. This function has an inverse.
 - f.** This function has at least three real zeros.
8. In function notation
 - a. $f(x)$ is the rule of the function.
 - b. $f(x)$ is a domain element.
 - c.** $f(x)$ is a range element.
 - d.** f is the name of the function.



Fig. 1

(2 pts each for 9 – 15)

9. The vertex of the graph of a quadratic function is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.
10. The absolute value inequality $|3x - 5| < 7$ is equivalent to the compound linear inequality **$-7 < 3x - 5 < 7$** .
11. To ask: "Where is $f(x) > 0$?" is the same as asking: "Where is the graph of f **above** the **x-axis**?"
12. When considering the three statements $2x - 5 < 0$, $2x - 5 = 0$, and $2x - 5 > 0$, the statement $2x - 5 = 0$ is called the **boundary equation**.
13. If f and g are functions for which $f \circ g(x) = x$ and $g \circ f(x) = x$, then f and g are **inverses** of each other.
14. A function has an inverse if and only if its graph passes the **horizontal** line test.
15. The point (a, b) is on the graph of the function f if and only if $b = \mathbf{f(a)}$.

In the following graphing exercises make sure you label the x-intercepts, y-intercepts, and vertices with their coordinates.

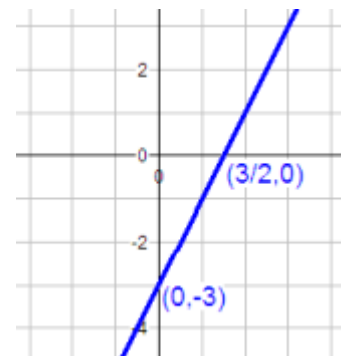
16. **(7 pts)** Sketch the graph of the function whose rule is $f(x) = 2x - 3$. Show all computations.

$f(0) = -3$. So $(0, -3)$ is the y-intercept.

$f(x) = 0$ implies $2x - 3 = 0$

The solution is $x = \frac{3}{2}$

So $\left(\frac{3}{2}, 0\right)$ is the x-intercept.



17. **(7 pts.)** Sketch the graph of the function whose rule is $f(x) = x^2 - 4x - 5$. Show all computations.

$$f(x) = x^2 - 4x - 5 = (x - 5)(x + 1)$$

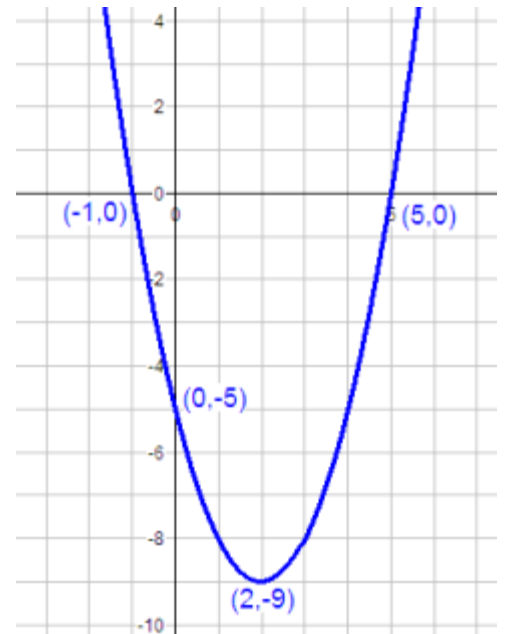
$$f(x) = 0 \text{ if and only if } x \in \{5, -1\}$$

The graph of f is a parabola which opens up.

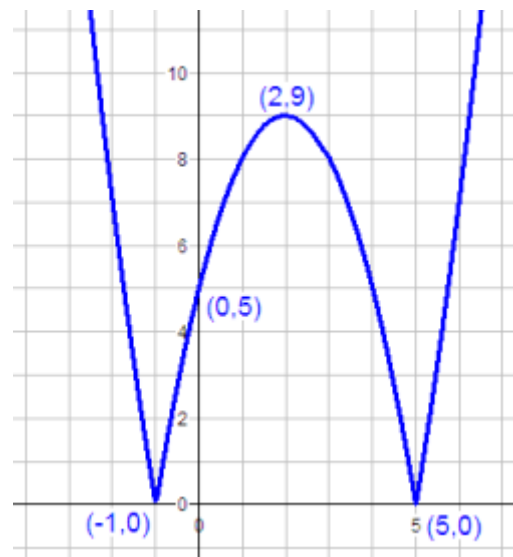
The y -intercept is $f(0) = -5$.

$$\text{The vertex is } \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right) \right) = \left(\frac{4}{2}, f(2) \right) = (2, -9)$$

$$\text{Note } f(2) = (2 - 5)(2 + 1) = (-3)(3) = -9$$



18. **(7 pts)** Sketch the graph of the function whose rule is $f(x) = |x^2 - 4x - 5|$. No computations required.



19. **(7 pts)** Solve the inequality $|2x - 5| > 7$. Write the solution set using interval notation.

Solve $|2x - 5| < 7$ which is equivalent to the compound inequality

$$\mathbf{-7 < 2x - 5 < 7}$$

$$\mathbf{-2 < 2x < 12}$$

$$\mathbf{-1 < x < 6}$$

The solution set for $|2x - 5| < 7$ is the interval $(-1, 6)$

From the Law of Trichotomy we then conclude the solution set for $|2x - 5| > 7$ is the union $(-\infty, -1) \cup (6, \infty)$

20. **(10 pts)** Consider the graph in Fig. 2. Use interval notation to answer Items b, c, and d. Use the roster method to answer Items a and e.

a. Where is $f(x) = 0$? $\{-2, 0, 2\}$

b. Where is $f(x) < 0$? $(-\infty, -2) \cup (0, 2)$

c. Where is $f(x) > 0$? $(-2, 0) \cup (2, \infty)$

d. Where is f decreasing? $(-k, k)$

e. What are the real zeros of f ? $\{-2, 0, 2\}$

21. **(7 pts.)** Find the inverse of the function whose rule is $f(x) = 5x - 8$. Clearly and neatly show all five steps.

$$\mathbf{y = 5x - 8}$$

$$\mathbf{x = 5y - 8}$$

$$\mathbf{x + 8 = 5y}$$

$$y = \frac{x + 8}{5}$$

$$f^{-1}(x) = \frac{x + 8}{5}$$

- a) Law of Trichotomy
- b) Transitive Property
- c) Distributive Property
- c-1) Definition of function
- d) Function notation
- e) Definition of graph
- f) Definition of zero of a function
- g) Zero Factor Property
- h) Convention when domain is not specified
- j) Rule for h
- k) Vertical line test
- m) Horizontal line test
- n) Definition of inverse of a function
- p) Definition of linear function
- q) Definition of quadratic function
- r) Definition of composition
- s) Definition of sum of functions
- t) Definition of product of functions

MATCHING: For each of the following statements one of the above items is the justification. Enter the correct letter in the blank preceding each of the following numbered statements. Some of the above lettered items might get used more than once and some will not be used. **(10 pts)** For this exercise let f , g , and h be functions whose rules are:

$$f(x) = 4x - 5 \quad g(x) = \frac{x+5}{4} \quad h(x) = \frac{x^2 - 4}{x - 5}$$

- _h_** 1) The domain of h is $(-\infty, 5) \cup (5, \infty)$.
- _e_** 2) Because $f(1) = -1$, the point $(1, -1)$ is on the graph of f .
- _s_** 3) $(f + g)(3) = f(3) + g(3)$
- _n_** 4) If two functions m and n are inverses of each other, then $m \circ n(4) = 4$
- _r_** 5) $f \circ g(x) = f(g(x))$
- _b_** 6) If two expressions represent the same quantity, the two expressions are equal.
- _b_** 7) Because $g(2) = \frac{2+5}{4} = \frac{7}{4}$ we conclude that $g(2) = \frac{7}{4}$
- _f_** 8) $f\left(\frac{5}{4}\right) = 0$, therefore $\frac{5}{4}$ is a zero of f .
- _j_** 9) $h(3a+k) = \frac{(3a+k)-4}{(3a+k)-5}$
- _a_** 10) If a number d is not a solution of the equation $3x + 2 = 5x - 9$ and d is not a solution of the inequality $3x + 2 > 5x - 9$, then d is a solution of the inequality $3x + 2 < 5x - 9$

(6 pts) Definition: A **function** consists of three things;

- A set called the **domain**
- A set called the **range**
- A **rule** which associates **each** element of the **domain** with a **unique** element of the range.

(4 pts)

