Meramec College Algebra TEST 3 Solution Fall 2009

NAME:____________________________________Score____________________/100

Please print **Average for in-class tests and quizzes __________________**

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION. NO WORK – NO CREDIT

Circle T or F, whichever is correct. (1 pts each)

1. **T** F The functions exp and ln are inverses.
2. T **F** Every square matrix has a multiplicative inverse.
3. **T** F The graph of a 5^{th} degree polynomial function has at least one x-intercept.
4. T **F** Every rational function has a horizontal asymptote.
5. **T** F For each exponential function, the y-intercept is (0,1).

In the following multiple choice questions, any number of choices may be correct. In each question at least one choice is correct. Circle ALL correct choices. (1 pt. for each choice.)

6. Consider the function whose rule is $f(x) = \dfrac{x+1}{x-3}$.
   a. The zeros of f are -1 and 3.
   b. The x-intercept of the graph of f is (-3,0).
   c. The x-intercept of the graph of f is (-1,0).
   d. The x-intercept of the graph of f is (3, 0).
   e. The x-axis is a horizontal asymptote of f.
   f. The line y = 1 is a horizontal asymptote of f.
   g. There is no horizontal asymptote for f.
   h. The line x = 3 is a vertical asymptote.
   i. The line x = -3 is a vertical asymptote.
   j. The domain of f is all real numbers.
   k. The domain of f is all real numbers except 3 and -1.
   l. The domain of f is all real numbers except 3.

7. Consider the exponential function exp base e.
   a. The x-intercept of exp is (1,0).
   b. The y-intercept of exp is (0,1).
   c. The function exp has no x-intercept.
   d. The function exp has no y-intercept.
   e. The exp function satisfies the vertical line test.
   f. The exp function satisfies the horizontal line test.
   g. The exp function has an inverse.
   h. The exp function has a vertical asymptote.

8. Suppose f is a polynomial function f whose leading term is $-3x^5$ and whose constant term is 5.
   a. The graph of f tries to cross the x-axis 5 times.
   b. The graph of f has at least one x-intercept.
   c. The graph of f might have no x-intercepts.
   d. As $x \to +\infty, f(x) \to +\infty$.
   e. As $x \to -\infty, f(x) \to +\infty$.
   f. As $x \to +\infty, f(x) \to -\infty$.
   g. As $x \to -\infty, f(x) \to -\infty$
   h. The graph of f might have a horizontal asymptote.
   i. The graph of f might have a vertical asymptote.
   j. A possible rational zero of f is $\frac{3}{5}$.
   k. A possible rational zero of f is $\frac{5}{3}$.

(2 pts each for 9 – 15)

9. The boundary equation for $3x^2 - 2x^3 < 7$ is $\mathbf{3x^2 - 2x^3 = 7}$.
10. A **matrix** is a rectangular array of numbers.
11. The determinant is a **function** whose domain is the set of **square matrices**and whose range is the **real numbers**.
12. Two systems of equations are **equivalent** systems if they have the same solution sets.
13. The exponential function base e is the function exp whose rule may be written in the form exp(x) = $\mathbf{e^x}$.
14. The logarithm function base e is the function which is the **inverse** of the function exp.
15. Two matricies are equal if they have the same **order** and their corresponding **entries** are equal.
16. **(5 pts)** Use the substitution method to solve the system of equations

$$\begin{cases} 2x+3y=9 \\ x-4y=-1 \end{cases} \to \begin{cases} 2x+3y=9 \\ x=4y-1 \end{cases} \to \begin{cases} 2(4y-1)+3y=9 \\ x=4y-1 \end{cases} \to \begin{cases} 11y=11 \\ x=4y-1 \end{cases}$$

$$\to \begin{cases} y=1 \\ x=4y-1 \end{cases} \to \begin{cases} y=1 \\ x=3 \end{cases}$$

The solution is the ordered pair (3,1).

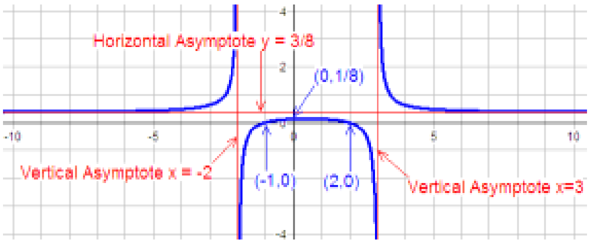
17. **(5 pts.)** Compute the determinant $\det\left(\begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}\right) = (2)(4)-(5)(-3) = 23$

18. **(5 pts.)** Sketch the graph of $f(x)=\dfrac{3(x-2)(x+1)}{8(x-3)(x+2)}$.

Zeros of the numerator are 2 and -1. They are real.
Zeros of the denominator are 3 and -2. They are real.
The domain is all real numbers except 3 and -2.
There are two vertical asymptotes: x = 3 and x = -2.
No zero of the numerator is a zero of the denominator. So (2, 0) and (-1, 0) are x-intercepts.
The degree of the numerator is equal to the degree of the denominator. Therefore the horizontal asymptote is $y=\frac{3}{8}$.

Tests: $f(0)=\dfrac{3(0-2)(0+1)}{8(0-3)(0+2)}>0$ $\quad f(4)=\dfrac{3(4-2)(4+1)}{8(4-3)(4+2)}>0$ $\quad f(-3)=\dfrac{3(-3-2)(-3+1)}{8(-3-3)(-3+2)}>0$

$$f\left(\frac{-3}{2}\right)=\frac{3(\frac{-3}{2}-2)(\frac{-3}{2}+1)}{8(\frac{-3}{2}-3)(\frac{-3}{2}+2)}<0 \qquad f\left(\frac{5}{2}\right)=\frac{3(\frac{5}{2}-2)(\frac{5}{2}+1)}{8(\frac{5}{2}-3)(\frac{5}{2}+2)}<0$$



The graph intesects it horizontal asymptote if and only if $f(x)=\frac{3}{8}$ so we solve $\dfrac{3(x-2)(x+1)}{8(x-3)(x+2)}=\dfrac{3}{8}$

Multiply both sides by the denominator to obtain

$3(x-2)(x+1)=\frac{3}{8}\left[8(x-3)(x+2)\right]$ which gives $(x-2)(x+1)=(x-3)(x+2)$

Multiply to get $x^2-x-2=x^2-x-6$ and simplify to $-2=-6$ which has no solution.

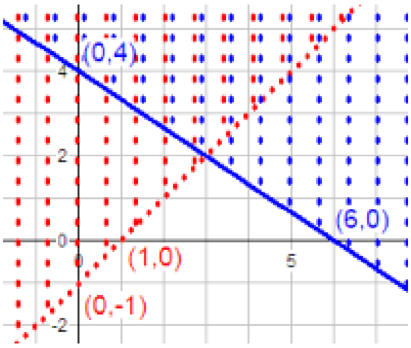
The solution set is $\varnothing$

Organize your facts here:

Kind of function: **Rational**	Domain: **All reals except 3 and -2**
x-intercepts are **(2, 0) and (-1, 0)**	Vertical asymptotes: **x=-2 and x = 3**
y-intercept is: $\left(0,\frac{1}{8}\right)$	Horizontal asymptotes: $y=\frac{3}{8}$
Test $f(0)=\frac{1}{8}>0$	Test $f\left(-\frac{3}{2}\right)<0$
Test **f(4) >0**	Test $f\left(\frac{5}{2}\right)<0$
Test **f(-3) >0**	

19. **(5 pts.)** Sketch the graph of the system $\begin{cases} x - y < 1 \\ 2x + 3y \geq 12 \end{cases}$

The boundary line for x − y < 1 is x − y = 1.
If x = 0, then y = -1, so (0, -1) is the y-intercept.
If y = 0, then x = 1, so (1, 0) is the x-intercept.
This should be a dashed line.



The boundary line for 2x + 3y ≥12 is 2x + 3y =12.
If x = 0, then y = 4, so (0, 4) is the y-intercept.
If y = 0, then x = 6, so (6, 0) is the x-intercept.
This should be a solid line.
Test (0,0) in x − y < 1 to obtain 0 − 0 < 1 which is true.
Test (0, 0) in 2x + 3y ≥12 to obtain 0 + 0 ≥12 which is false.

20. **(5 pts.)** Perform the multiplication:

$$\begin{bmatrix} 1 & 2 & -4 \\ -2 & -3 & 3 \end{bmatrix} \begin{bmatrix} 2 & -2 \\ 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \boxed{12} & \boxed{-8} \\ \boxed{-16} & \boxed{7} \end{bmatrix}$$

21. **(5 pts.)** What are the possible rational zeros of f(x) = $4x^5 + 8x^3 + 4x - 3$.

$$p \in \{\ \pm 1,\ \pm 3\}$$

$$q \in \{\pm 1,\ \pm 2,\ \pm 4\}$$

$$\frac{p}{q} \in \left\{\pm 1,\ \pm 3,\ \pm \frac{1}{2},\ \pm \frac{3}{2},\ \pm \frac{1}{4},\ \pm \frac{3}{4}\right\}$$

22. **(5 pts.)** Supply the missing entries by performing the indicated elementary row operation.

$$\begin{bmatrix} 2 & 1 & \frac{1}{5} & 0 \\ -4 & -3 & 0 & 1 \end{bmatrix} \xrightarrow{2R_1+R_2 \longrightarrow R_2} \begin{bmatrix} \boxed{2} & \boxed{1} & \boxed{\frac{1}{5}} & \boxed{0} \\ \boxed{0} & \boxed{-1} & \boxed{\frac{2}{5}} & \boxed{1} \end{bmatrix}$$

23. **(5 pts.)** Consider the matrices. A = $\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$ X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ C = $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and

$$\boldsymbol{A}^{-1} = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix}$$

Solve the matrix equation AX = C.

Multiply both sides by A^{-1} to obtain the solution X = $A^{-1}C$

So the solution is $X = \begin{bmatrix} -13 & 6 & 4 \\ 12 & -5 & -3 \\ -5 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 29 \\ -25 \\ 10 \end{bmatrix}$

24. **(5 pts.)** Solve the equation $e^{5x-3} = 2$

$$e^{5x-3} = 2$$

$$\exp(5x-3) = 2$$

$$\ln(\exp(5x-3)) = \ln(2)$$

$$5x - 3 = \ln(2)$$

$$x = \frac{3 + \ln(2)}{5}$$

25.

(5 points) You do not need to do any computations. Simply fill in the blanks to describe the process for finding the inverse of a matrix.

To find the inverse of the matrix A = $\begin{bmatrix} 5 & 7 & 4 \\ 3 & -1 & 3 \\ 6 & 7 & 5 \end{bmatrix}$

Begin by adjoining the **identity** matrix to obtain the matrix $\begin{bmatrix} 5 & 7 & 4 & 1 & 0 & 0 \\ 3 & -1 & 3 & 0 & 1 & 0 \\ 6 & 7 & 5 & 0 & 0 & 1 \end{bmatrix}$ with order **3X6**

The next step is to get a **1** in the**11** position.

Then use that **1** to get **0** everywhere else in the **first column**

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 7/5 & 4/5 & 1/5 & 0 & 0 \\ 0 & -26/5 & 3/5 & -3/5 & 1 & 0 \\ 0 & -7/5 & 1/5 & -6/5 & 0 & 1 \end{bmatrix}$

The next step is to get a **1** in the **22** position.

Then use that **1** to get **0** everywhere else in the **second column**

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 0 & 25/26 & 1/26 & 7/26 & 0 \\ 0 & 1 & -3/26 & 3/26 & -5/26 & 0 \\ 0 & 0 & 1/26 & -27/26 & -7/26 & 1 \end{bmatrix}$

The next step is to get a **1** in the **33** position.

Then use that **1** to get **0** everywhere else in the **third column**

At this point the matrix will have been converted to $\begin{bmatrix} 1 & 0 & 0 & 26 & 7 & -25 \\ 0 & 1 & 0 & -3 & -1 & 3 \\ 0 & 0 & 1 & -27 & -7 & 26 \end{bmatrix}$

The inverse of A is the matrix $A^{-1} = \begin{bmatrix} 26 & 7 & -25 \\ -3 & -1 & 3 \\ -27 & -726 & \end{bmatrix}$

Which has order **3X3**