Meramec College Algebra TEST 1 Solution Summer 2009

NAME: _____ Score _____/100
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SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Circle T or F, whichever is correct. **2 pts. each for 1 – 25. 5 pts. each for all others.**

1. T **F** A solution of the equation $x^3 - x^2 - x - 3 = 0$ is 2.

2. T **F** $(-2, 5] = \{x | x \in R \text{ and } -2 \le x < 4\}$

3. T **F** The graph of a linear equation in one variable is a line.

4. T **F** The complex component of $7 - 8i$ is $-8i$.

5. **T** F The sum of a complex number and its conjugate is a real number. (Hint:Do it)

6. **T** F $3 + 9 = 17$ is a mathematical equation.

Circle the symbol for the smallest set of numbers which contains the number given at the left.

The Symbols are standard: **C** is the complex numbers, **R** is the real numbers, **F** is the irrational numbers, **Q** is the rational numbers, **Z** is the integers, **W** is the whole numbers, and **N** is the natural numbers.

7. The smallest set which contains -3 is **C R F Q Z W N**

8. The smallest set which contains $\frac{2}{5} - 7i$ is **C R F Q Z W N**

9. The smallest set which contains 8 is **C R F Q Z W N**

10. The smallest set which contains $\frac{2}{3}$ is **C R F Q Z W N**

11. The smallest set which contains $\sqrt{7}$ is **C R F Q Z W N**

12. The smallest set which contains $-4i$ is **C R F Q Z W N**

Fill in each of the blanks to make the statements true.

13. The exact volume of a cylinder with radius 5 and height 3 is $V = \pi r^2 h = \pi(5^2)(3) = 75\pi$.

14. Write the formula for the area of a triangle. $A = \frac{1}{2}bh$. **(Your answer must be a formula.)**

15. A linear equation in one variable is an equation which may be written in the form $ax + b = 0$.

16. A quadratic equation in one variable is an equation which may be written in the form $ax^2 + bx + c = 0$.

17. A complex number is a number that can be written in the form $a + bi$ where a and b are real numbers and $i = \sqrt{-1}$.

18. A number (or numbers) that makes an equation **true** when substituted for the variable (or variables) is called a **solution** of the equation.

19. If any expression is added to both sides of an equation the resulting equation is **equivalent** to the original equation.

20. The norm of 2 – 5i is $\mathbf{2^2 + (-5)^2 = 4 + 25 = 29}$.

21. The discriminant of $2x^2 - 3x + 4$ is $\mathbf{b^2 - 4ac = (-3)^2 - 4(2)(4) = 9 - 32 = -23}$.

Circle <u>all</u> the words which could be used to correctly complete the sentence.

22. 2x – 7 = 0 is a (**linear** quadratic identity **conditional** contradiction) equation.

23. 2x – 5 = 2x + 3 is a (**linear** quadratic identity conditional **contradiction**) equation.

24. (x+2)(x + 5) = x^2 + 7x + 10 is a (linear **quadratic** **identity** conditional contradiction) equation.

25. $3x^2$ + 4x = 3x + 2 is a (linear **quadratic** identity **conditional** contradiction) equation.

26. Write the quadratic formula.

The solutions of the quadratic equation $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

27. Write the multiplicative inverse of the complex number 5 – 3i.
The multiplicative inverse is the conjugate divided by the norm. So in this case the multiplicative inverse is
$\frac{5+3i}{5^2+3^2} = \frac{5+3i}{34}$

28. Write the zero factor property. **If a and b are real numbers and if ab = 0, then a = 0 OR b = 0.**

29. Sketch the graph of 2x – 6 = 0.

30. Compute the product (3 + 2i)(1 – 5i). Your answer should be written in <u>standard form</u>. **(Show your work. no work: no credit)**

$(3 + 2i)(1 - 5i) = 3 - 15i + 2i - 10i^2 = 3 - 13i + 10 = 13 - 13i.$

31. Change (3 + 2i) ÷ (1 – 5i) into an equivalent multiplication problem. **(D0 NOT perform the multiplication.)**

$$3 + 2i \quad \div \quad (1 - 5i)$$
$$\downarrow \quad \downarrow \quad \downarrow$$
$$3 + 2i \quad \cdot \quad \left(\frac{1+5i}{26}\right)$$

32. Use the Quadratic formula to solve the equation $x^2 + x - 1 = 0$. **(Show your work. no work: no credit)**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - (4)(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$

33. Solve the equation $3x - 2 = \sqrt{3}\,x + 8$. **(Show your work. no work: no credit)**

$3x - 2 = \sqrt{3}x + 8$

$3x - 2 - \sqrt{3}x = 8$

$3x - \sqrt{3}x = 10$

$(3 - \sqrt{3})x = 10$

$$x = \frac{10}{3 - \sqrt{3}}$$

34. A cone has radius 5 feet and volume 25π cubic feet. What is the height? **(Show your work. no work: no credit)**

Use $V = \frac{1}{3}\pi r^2 h$

$$25\pi = \frac{1}{3}\pi\left(5^2\right)h$$

$75\pi = 25\pi h$

$$h = \frac{75\pi}{25\pi} = 3$$

38. Two cyclists, 108 mi apart start riding toward each other at the same time. One cycles twice as fast as the other. If they meet 2 hours later, at what average speed is each cyclist traveling?
Fill in the blanks to provide a correct and complete analysis of this question.

Solution:

Let x be the speed of the slower cyclist.

The speed of the other cyclist is **2x**.

The distance traveled by the slower cyclist (in 2 hr.) is **2x**.

The distance traveled by the second cyclist (in 2 hr.) is **4x**.

The sum of the distance traveled by the two cyclists is **2x + 4x = 6x**.

The sum of the distance traveled by the two cyclists is **108**. **Must be different than the previous blank.**

We now have two expressions for the **sum of the distance traveled by the two cyclists**.

The model is therefore 6x = **108**.

Ordinary means to solve this equation produces x = 18.

Conclusion: The slower cyclist is traveling at an average speed of **18 mph** and the other at **36 mph**.