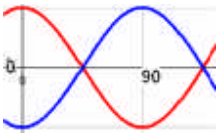
PreCalculus Exercises Section 6.3 Page 502

33. Is $\sin(4x) = 4\sin(x)\cos(x)$ an identity?
Discussion: Begin by testing an example.
Let $x = \pi$.
Then $\sin(4x) = \sin(4\pi) = \sin(0) = 0$.
Also note that when $x = \pi$,
then $4\sin(x)\cos(x) = 4\sin(\pi)\cos(\pi) = 4(0)(-1) = 0$
It now follows that $\sin(4x) = 4\sin(x)\cos(x)$.
Sometimes this conclusion is affirmed by stating that $x = \pi$ provides a counterexample to the claim.

34. Is $\csc(2x) = 2\csc(x)\sec(x)$ an identity?
Discussion: Begin by using a calculator to test an exa
Let $x = 15°$
Then $\csc(2x) = \csc(20°) = \dfrac{1}{\sin(20^\circ)} = 2.924$
and $2\csc(x)\sec(x) = 2\csc(10)\sec(10)$
$= 2\dfrac{1}{\sin(10^\circ)\cos(10^\circ)} = 0.342$
It now follows that $\csc(2x) = 2\csc(x)\sec(x)$ is not an identity.

37. Is $\cos(2x) = 1 - 2\cos^2(x)$?
Discussion: Begin by using a graphing utility to compare the graphs of the two functions $f(x) = \cos(2x)$ and $g(x) = 1 - 2\cos^2(x)$.
In Figure 37, the graph of f is shown in red and the graph of g is shown in blue. Clearly, for many values of x the range values are not equal.



It now follows that $\cos(2x) = 1 - 2\cos^2(x)$ is not an identity. If a specific counterexample is desired, $x = 90°$ would provide such a counterexample. (Now look at 37B)

37B. Is $1 - \cos(2x) = \tan(x)\sin(2x)$?
Discussion: Begin by using a graphing utility to compare the graphs of the two functions $f(x) = 1 - \cos(2x)$ and $g(x) = \tan(x)\sin(2x)$.
In Figure 37B, the graph of f is shown in blue and the graph of g is shown in red. It appears that for every value of x, the range values are equal. This indicates that we are probably considering an identity. Therefore we should attempt to construct a proof rather than find a counterexample. The picture may be a good indicator, but it does not constitute a proof.

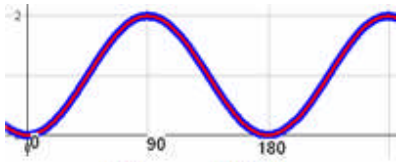


Figure 37B

Proof:
$$1 - 2\cos(2x) = 1 - (1 - 2\sin^2(x)) = 2\sin^2(x)$$
$$= \left(2\sin^2(x)\right)\left(\frac{\cos(x)}{\cos(x)}\right)$$
$$= \left(\frac{\sin(x)}{\cos(x)}\right)\left(2\sin(x)\cos(x)\right)$$
$$= \tan(x)\sin(2x)$$