

Name _____ Score _____/10

Please Print Clearly1. Consider the polynomial function whose rule is $f(x) = (x - 4)^2(x + 2)^4(x - 1)^6$ a) What are the zeros of f and what are their multiplicities? **Zeros of the function and their multiplicities are numbers.****4 is a zero with multiplicity 2 -2 is a zero with multiplicity 4 1 is a zero with multiplicity 6**b) Present an argument which establishes that the graph of f is never below the x -axis.**Each factor of $f(x)$ is raised to an even power and is therefore non-negative. The product of three non-negative factors is non-negative. Therefore $f(x)$ is non-negative and it follows that the graph of f is never below the x -axis.****The multiplicities of each of the zeros is an even number.****Therefore the graph does not cross the axis.** **$f(0) = (-4)2(2)4(-1)6$ which clearly is positive so the point $(0, f(0))$ is above the x -axis.****These two underlined facts imply that the graph of f is never below the x -axis.****Comment: Both facts are required.**2. Suppose g is a polynomial function with odd degree. Explain why the graph of f must cross the x -axis at least once.**A polynomial function has exactly the same number of zeros as its degree. Complex zeros occur in conjugate pairs. Therefore a polynomial of odd degree must have an odd number of real zeros and at least one of them must have odd multiplicity. A real zero with odd multiplicity corresponds with an x -intercept where the graph crosses the x -axis.**3. Answer the following questions about the **rational** function whose graph is shown at the right.a) What are the apparent real zeros of the numerator of this rational function? **Zeros of the numerator are numbers.****It appears that the real zeros are 0 and 1.**b) What are the apparent real zeros of the denominator of this rational function? **Zeros of the denominator are numbers.****There are none. The graph seems to indicate the domain of f is all real numbers.**c) What is the apparent horizontal asymptote of this rational function? **Give the equation for the line.****It appears that the line $y = 0$ (the x -axis) is the horizontal asymptote.**

d) Compare the degree of the numerator with the degree of the denominator for this rational function?

Because the line $y = 0$ is a horizontal asymptote, the degree of the numerator is less than the degree of the denominator.e) **(was omitted)** What is the degree of the numerator of this rational function?**If the numerator has no complex zeros then its degree is 2. If the numerator has complex zeros then its degree is an even number less than the degree of the denominator.**