SLU PreCalculus Quiz 6 Solution Fall 2007

NAME: ____ Score________________________________/10

Please **print** your name

1.Prove $\tan^2\alpha - \sin^2\alpha = \tan^2\alpha\sin^2\alpha$

Proof 1:

$$\tan^2\alpha - \sin^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} - \sin^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} - \frac{\sin^2\alpha\cos^2\alpha}{\cos^2\alpha} = \frac{\sin^2\alpha\left(1-\cos^2\alpha\right)}{\cos^2\alpha} = \left(\frac{\sin^2\alpha}{\cos^2\alpha}\right)\left(\sin^2\alpha\right) = \tan^2\alpha\sin^2\alpha$$

Proof 2:

$$\tan^2\alpha\sin^2\alpha = \left(\sec^2\alpha - 1\right)\sin^2\alpha = \sec^2\alpha\sin^2\alpha - \sin^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} - \sin^2\alpha = \tan^2\alpha - \sin^2\alpha$$

Proof 3:

$$\tan^2\alpha\sin^2\alpha = \tan^2\alpha\left(1-\cos^2\alpha\right) = \tan^2\alpha - \tan^2\alpha\cos^2\alpha = \tan^2\alpha - \left(\frac{\sin^2\alpha}{\cos^2\alpha}\right)\left(\cos^2\alpha\right) = \tan^2\alpha - \sin^2\alpha$$

Proof 4:

$$\tan^2\alpha\sin^2\alpha = \left(\frac{\sin^2\alpha}{\cos^2\alpha}\right)\sin^2\alpha = \frac{\left(1-\cos^2\alpha\right)\sin^2\alpha}{\cos^2\alpha} = \frac{\sin^2\alpha - \sin^2\alpha\cos^2\alpha}{\cos^2\alpha} = \frac{\sin^2\alpha}{\cos^2\alpha} - \frac{\cos^2\alpha\sin^2\alpha}{\cos^2\alpha} = \tan^2\alpha - \sin^2\alpha$$

2. Prove $\sin\beta = \dfrac{\tan\beta\cot\beta}{\csc\beta}$

Proof 1: $\dfrac{\tan\beta\cot\beta}{\csc\beta} = \dfrac{\tan\beta\left(\dfrac{1}{\tan\beta}\right)}{\dfrac{1}{\sin\beta}} = \dfrac{1}{\dfrac{1}{\sin\beta}} = \sin\beta$

Proof 2: $\dfrac{\tan\beta\cot\beta}{\csc\beta} = \dfrac{\left(\dfrac{\sin\beta}{\cos\beta}\right)\left(\dfrac{\cos\beta}{\sin\beta}\right)}{\dfrac{1}{\sin\beta}} = \dfrac{1}{\dfrac{1}{\sin\beta}} = (1)\left(\dfrac{\sin\beta}{1}\right) = \sin\beta$