SLU PreCalculus Quiz 7 Solution Fall 2007

NAME: ______________________________ Score ______________________ /10

Please **print** your name

1. Solve the equation 2 cos(x) = 1 for $0 \le x \le 2\pi$

2cos(x) = 1

$$\cos(x) = \frac{1}{2}$$

$$x = \cos^{-1}(\cos(x)) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

From which it follows that $|\cos(x)| = \frac{1}{2}$ **also at** $x = \frac{2\pi}{3}, \frac{4\pi}{3},$ **and** $\frac{5\pi}{3}$

However cos(x) > 0 only in Quadrants I and IV, so the solutions of the equation are $\frac{\pi}{3}$ **and** $\frac{5\pi}{3}$.

2. The solution set for $\sin^2(x) = \frac{1}{2}\sin(2x)$ for $0 \le x < 2\pi$ is $\left\{0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}\right\}$

What are **all** solutions for $\sin^2(x) = \frac{1}{2}\sin(2x)$

$$x = \begin{cases} 0 + 2k\pi \\ \frac{\pi}{4} + 2k\pi \\ \pi + 2k\pi \\ \frac{5\pi}{4} + 2k\pi \end{cases}$$

If you examine several numbers in each list duplications are observed and the solutions can be enumerated in a more efficient manner.

$$\left.\begin{matrix} 0 + 2k\pi \longrightarrow 0,\ 2\pi,\ 4\pi,\ 6\pi \\ \pi + 2k\pi \longrightarrow \pi,\ 3\pi,\ 5\pi,\ 7\pi \end{matrix}\right\} \longrightarrow 0,\ \pi,\ 2\pi,\ 3\pi,\ 4\pi,\ 5\pi,\ 6\pi,\ 7\pi\ \cdots \longrightarrow 0 + k\pi \longrightarrow k\pi$$

So these two cases collapse into the <u>set of all multiples of π</u>

Now examine the last two cases.

$$\left.\begin{matrix} \frac{\pi}{4} + 2k\pi \longrightarrow \frac{\pi}{4}, \frac{9\pi}{4}, \frac{17\pi}{4}, \frac{25\pi}{4} \\ \frac{5\pi}{4} + 2k\pi \longrightarrow \frac{5\pi}{4}, \frac{13\pi}{4}, \frac{21\pi}{4}, \frac{29\pi}{4} \end{matrix}\right\} \longrightarrow \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}, \frac{25\pi}{4} \longrightarrow \frac{\pi}{4} + 2k\pi$$

So these two cases collapse into the set of real numbers of the form $\frac{\pi}{4}$ <u>plus even multiples of π</u>.

Please observe that this last collection cannot be described as multiples of $\frac{\pi}{4}$