SLU PreCalculus Quiz 9 Solution Fall 2007

NAME: ______________________________ Score____________________/10

Please **print** your name

1. Solve the triangle in Fig. 1.

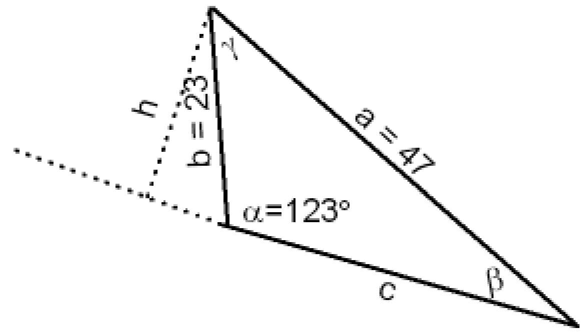

Fig. 1

h = 23 sin 57° = 19.29 < 47 = a.

Therefore there is the possibility of two triangles.

Use the Law of Sines to solve for β_1.

$$\frac{\sin\alpha}{a} = \frac{\sin\beta_1}{b} \Rightarrow \sin\beta_1 = b\left(\frac{\sin\alpha}{a}\right) = 23\left(\frac{\sin 123^\circ}{47}\right)$$

$$\beta_1 = \sin^{-1}(\sin\beta) = \sin^{-1}\left(23\left(\frac{\sin 123^\circ}{47}\right)\right) = 24.23^\circ$$

$$\gamma_1 = 180^\circ - 24.23^\circ - 123^\circ = 32.77^\circ$$

The second possible value for β is in the second quadrant

and is $\beta_2 = 180^\circ - 24.23^\circ = 155.77^\circ$

However, because 155.77 + 123 > 180, this value for β_2 does not yield a triangle.

For $\beta = 24.23^\circ$ and $\gamma = 32.77$ the Law of Sines may be used to determine c.

$$c = (\sin\gamma)\left(\frac{a}{\sin\alpha}\right) = (\sin 32.77)\left(\frac{47}{\sin 123}\right) = 30.33$$

2. Solve the triangle in Fig. 2

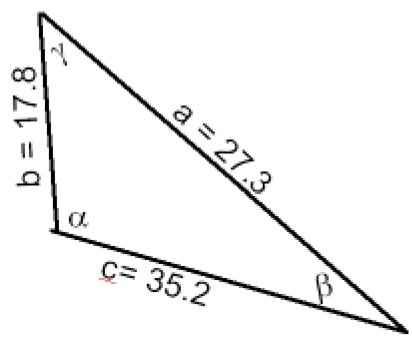

Fig. 2

From the Law of Cosines

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} \text{ and then } \alpha = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$

$$= \cos^{-1}\left(\frac{17.8^2 + 35.2^2 - 17.8^2}{(2)(17.8)(35.2)}\right) = 49.69^\circ$$

Similarly

$$\beta = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) = \cos^{-1}\left(\frac{27.3^2 + 35.2^2 - 17.8^2}{(2)(27.3)(35.2)}\right) = 29.82^\circ$$

and

$$\gamma = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right) = \cos^{-1}\left(\frac{27.3^2 + 17.8^2 - 35.2^2}{(2)(27.3)(17.8)}\right) = 100.49^\circ$$