

NAME: _____ Score _____ /100
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SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Questions 1 – 15 are each worth 2 points. Questions 16 – 25 are each worth 5 points. The last three questions combined are worth 20 points.

Circle T or F, whichever is correct.

1. T **F** Every function has an inverse.
2. T **F** To verify (prove) that two functions f and f^{-1} are inverses of each other it is necessary to show that both of the following are true:
 $f^{-1} \circ f(x) = x$ for all x in the domain of f and
 $f \circ f^{-1}(x) = x$ for all x in the domain of f^{-1}
3. T **F** If the same thing is done to both sides of an equation the resulting equation is equivalent to the original equation.
4. T **F** A zero of a function is an element of the range.
5. T **F** The rule for any function is expressed as an equation.
6. T **F** The product of two functions is a real number.

Fill in each of the blanks to make the statements true.

7. The **graph** of a function is the set of all points whose coordinates are $(a, f(a))$ where a is an element of the domain.
8. The domain of the \ln function is **all positive real numbers** $(0, \infty)$ $\{x \mid x > 0\}$
9. The composition of a function f with a function g is a function named **$f \circ g$** whose rule is
 $f \circ g(x) = f(g(x))$

10. Absolute value is defined by $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

11. If a horizontal line may be drawn so that it intersects the graph of a function in more than one point, then the function **does not** have an inverse.
12. The graph of a function is the set of all points of the form $(a, f(a))$ where a is an element of the domain and $f(a)$ is the corresponding range element..
13. The line which passes through $(0,5)$ with slope 14 is the graph of the **linear** function whose rule is $f(x) = 14x + 5$
14. The graph of the function whose rule is $f(x) = -3x^2 + 2x - 98$ is a **parabola** which opens **down**.
15. The slope of the line through the points $(-3, 2)$ and $(7, -5)$ is $-\frac{7}{10}$

For Problems 16 – 25, Show all necessary work. NO WORK – NO CREDIT
Sentences are good

16. What is the domain of the function whose rule is $f(x) = \frac{3x + 4}{\sqrt{x - 5}}$?

By convention, the domain of f is the largest set of real numbers for which the rule makes sense. Therefore the domain of f is the set of real numbers for which the denominator is not zero and for which the radical is a real number. Therefore the domain of f is all real numbers for which $x - 5 > 0$.
The domain of f is $(5, \infty)$

17. The rule for a function f is given by the equation $f(x) = \frac{x-3}{2x}$ and the rule for a function g is given by the equation $g(x) = x - 2$. Determine the rule for the function $f \circ g$.

$$f \circ g(x) = f(g(x)) = f(x-2) = \frac{(x-2)-3}{2(x-2)} = \frac{x-5}{2x-4}$$

18. Suppose f and g are functions whose rules are $f(x) = x - 3$ and $g(x) = \frac{1}{x^2}$. Calculate $f \circ g(2)$.

$$f \circ g(2) = f(g(2)) = f\left(\frac{1}{2^2}\right) = \left(\frac{1}{4}\right) - 3 = \frac{1-12}{4} = -\frac{11}{4}$$

19. Find the inverse of the function whose rule is $f(x) = \frac{3}{5}x + \frac{7}{4}$.

$$y = \frac{3}{5}x + \frac{7}{4}$$

$$x = \frac{3}{5}y + \frac{7}{4}$$

$$x - \frac{7}{4} = \frac{3}{5}y$$

$$\frac{5}{3}\left(x - \frac{7}{4}\right) = y$$

$$y = \frac{5}{3}x - \frac{35}{12}$$

$$f^{-1}(x) = \frac{5}{3}x - \frac{35}{12}$$

20. Suppose f and g are functions whose rules are $f(x) = 2x - 3$ and $g(x) = \frac{1}{3}x - 2$.

Verify (prove) that f and g are **not** inverses of each other.

$$f \circ g(x) = f\left(\frac{1}{3}x - 2\right) = 2\left(\frac{1}{3}x - 2\right) - 3 = \frac{2}{3}x - 4 - 3 \neq x$$

21. Find the rule (equation) for the linear function whose graph is perpendicular to the graph of $3x + 4y = 8$ and contains the point $(3,5)$. Use function notation to state the rule.

Because the desired function is linear its rule has the form $f(x) = mx + b$

Write $3x + 4y = 8$ in slope-intercept form $y = -\frac{3}{4}x + 2$ to see that its slope is $-\frac{3}{4}$.

Therefore the slope of the desired linear function is $\frac{4}{3}$.

The rule for the desired function has the form $f(x) = \frac{4}{3}x + b$

Because the point $(3,5)$ is on the graph $f(3) = 5$

$$\text{However, } f(3) = f(x) = \frac{4}{3}(3) + b = 4 + b$$

It follows that $4 + b = 5$ and $b = 1$.

The rule for the desired function is $f(x) = \frac{4}{3}x + 1$

22. Solve the equation $e^{2x-1} = 405$. I want an exact solution –not an approximation –no decimals

$$e^{2x-1} = 405$$

$$\exp(2x - 1) = 405$$

$$\ln(\exp(2x - 1)) = \ln(405)$$

$$2x - 1 = \ln(405)$$

$$x = \frac{1 + \ln(405)}{2}$$

23. Solve the equation $\ln(x) - \ln(5) = \ln(2) - \ln(x-3)$

$$\ln(x) - \ln(5) = \ln(2) - \ln(x-3)$$

$$\ln\left(\frac{x}{5}\right) = \ln\left(\frac{2}{x-3}\right)$$

$$\exp\left(\ln\left(\frac{x}{5}\right)\right) = \exp\left(\ln\left(\frac{2}{x-3}\right)\right)$$

$$\frac{x}{5} = \frac{2}{x-3}$$

$$x(x-3) = 10 \quad \text{might not be equivalent to the previous equations}$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

By The Zero Factor Property

$$x - 5 = 0 \text{ OR } x + 2 = 0$$

$$x = 5 \text{ OR } x = -2$$

Test 5: $\ln(x) = \ln(5)$ is defined

$$\ln(x-3) = \ln(5-3) = \ln(2) \text{ is defined}$$

Therefore 5 is a solution.

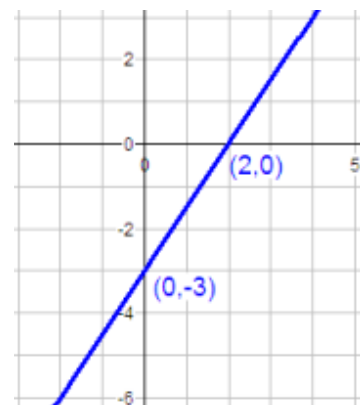
Test -2: $\ln(x) = \ln(-2)$ is not defined.

Therefore -2 is not a solution

24. Sketch the graph of the function whose rule is $f(x) = \frac{3}{2}x - 3$. Label the x and y intercepts with their coordinates

The y-intercept is -3.

$$f(x) = 0 \Rightarrow x = 2 \Rightarrow \text{the x-intercept is } 2$$



25. Sketch the graph of the function whose rule is $f(x) = 2x^2 - 5x - 3$

Label the x and y intercepts and the vertex with their coordinates. Factoring helps.

$$f(x) = 2x^2 - 5x - 3 = (2x + 1)(x - 3)$$

Because f is a quadratic function, its graph is a parabola.

Because its leading coefficient is positive, the parabola opens up.

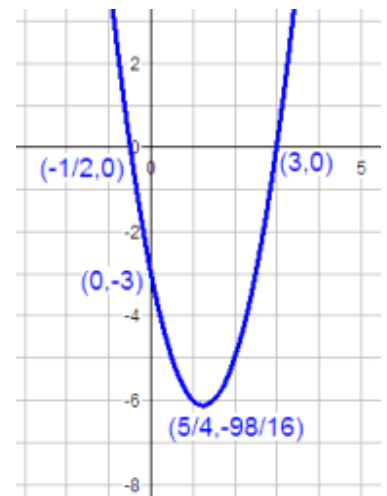
Clearly the zeros of f are the real numbers 3 and $-1/2$.

The x-intercepts are 3 and $-1/2$.

The vertex is

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \left(\frac{5}{4}, f\left(\frac{5}{4}\right)\right) = \left(\frac{5}{4}, 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) - 3\right)$$

$$\left(\frac{5}{4}, \frac{50}{16} - \frac{25}{4} - 3\right) = \left(\frac{5}{4}, \frac{50 - 100 - 48}{16}\right) = \left(\frac{5}{4}, -\frac{98}{16}\right)$$

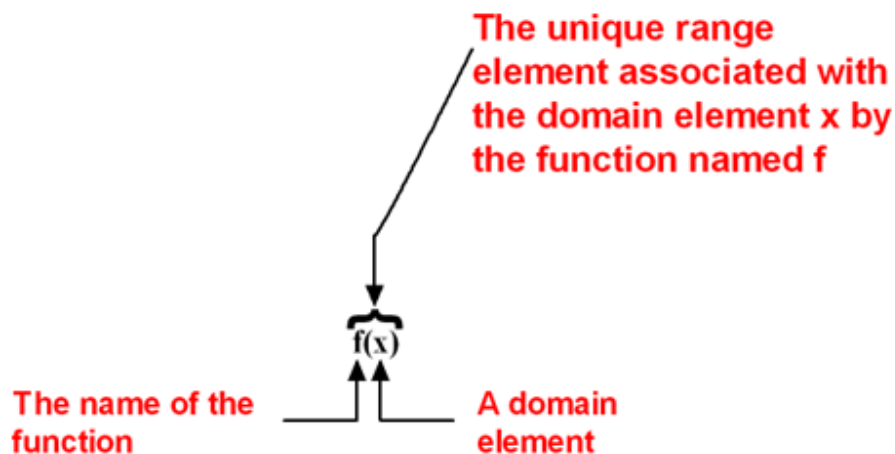


26. (6 points) Fill in the blanks

Definition: A **function** consists of three things

- A set called the **domain**
- A set called the **range**
- A **rule** which associates **each** element of the domain with a **unique** element of the range.

27. (4 points)



28. (10 points) Rules for functions are given at the top of the page and graphs of functions are given below them. Match the graphs and the rules by writing the letter which identifies a graph in the blank preceding a rule for a function.

a. **A** $f(x) = \frac{1}{x}$

b. **E** $f(x) = -3x + 3$

c. **F** $f(x) = -x^2$

d. **B** $f(x) = 3x + 3$

e. **C** $f(x) = x^2$

f. **I** $f(x) = (x + 2)^2$



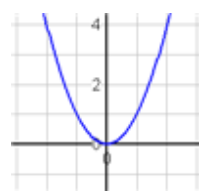
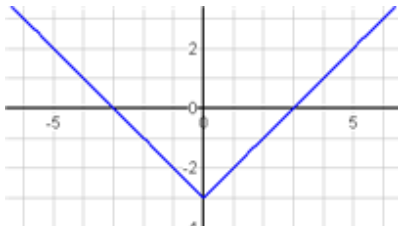

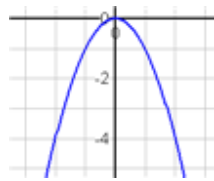

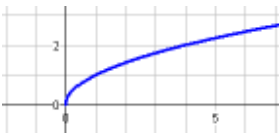
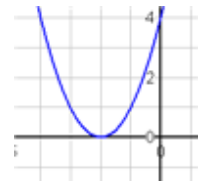
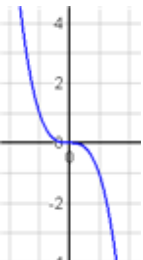

g. **D** $f(x) = |x| - 3$

h. **H** $f(x) = \sqrt{x}$

k. **K** x^3

m. **G** $\exp(x) = e^x$

The graphs are shown in blue.

<p>A.</p> 	<p>B.</p> 	<p>C.</p> 
<p>D.</p> 	<p>E.</p> 	<p>F.</p> 
<p>G.</p> 	<p>H.</p> 	<p>I.</p> 
<p>J.</p> 	<p>K.</p> 	<p>L.</p> 