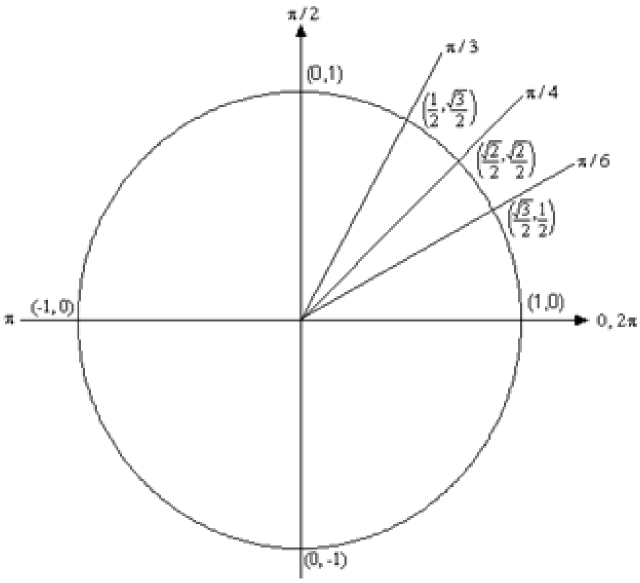
SLU PreCalculus TEST 2 Fall 2007

NAME: ____ Score______________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Questions are each worth 4 points.



1. $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

2. What is the exact value of cos(150°)? $-\frac{\sqrt{3}}{2}$

3. Circle those values in the following list which are zeros of the sin function.

   $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}$

4. Circle those values in the following list which are zeros of the cos function.

   $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}$

5. If sin(θ) < 0 and tan(θ) < 0, then θ is in Quadrant **IV**.

6. Find the exact value of tan(240°). $\sqrt{3}$

7. $\sin^{-1}(120°)$ is **undefined** .

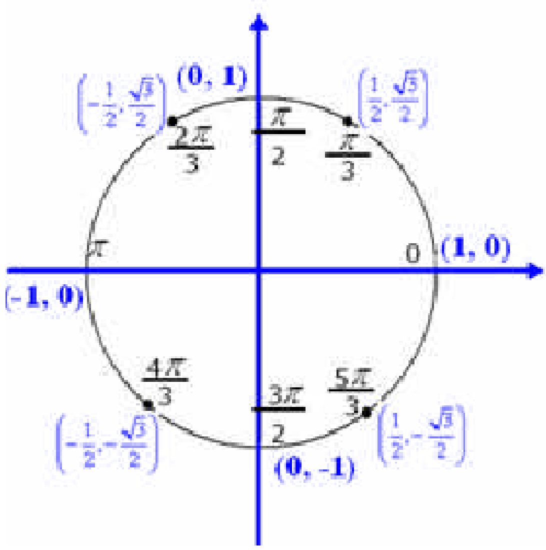
8. In a right triangle with acute angles $\alpha = \frac{3\pi}{8}$ and β,

   the exact value of β is $\frac{\pi}{8}$

9. What is the amplitude of the 3sin(5x + $\frac{\pi}{3}$) ? **3**

10. If W is the wrapping function, $W\left(\frac{2\pi}{3}\right)$ is a point P on the unit circle.

    Plot that point P on the unit circle at the right and label it with its coordinates.



The unit circle at the right contains more than the point requested. You should have plotted the point labeled $\frac{2\pi}{3}$ with the coordinates $\left(-\frac{1}{2}, \frac{\sqrt{3}}{3}\right)$

For Problems 11 – 25, Show all necessary work. NO WORK – NO CREDIT
Sentences are good

11. In Figure 1 at the right, b = 24, a = 45, and c = 51. Calculate sin(β). Write your answer as a fraction reduced to lowest terms.

$$\sin(\beta) = \frac{Opp}{Hyp} = \frac{b}{c} = \frac{24}{51} = \frac{8}{17}$$

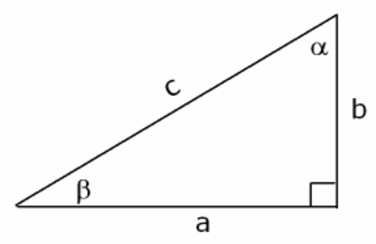


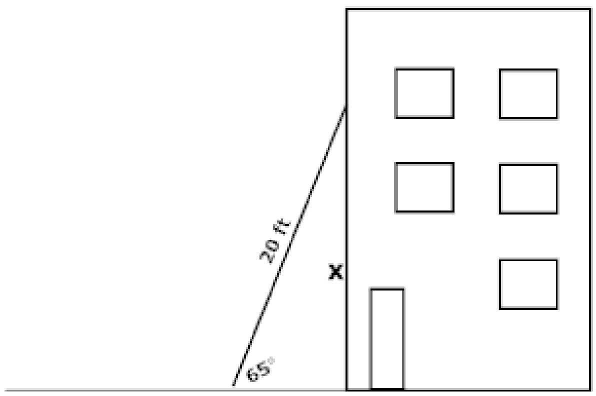
Figure 1

12. If a 20 foot ladder leans against a building such that it makes an angle of 65° with the ground, how high does the ladder reach on the building? Give your answer correct to the nearest foot. Draw a picture.

With reference to the diagram at the right,

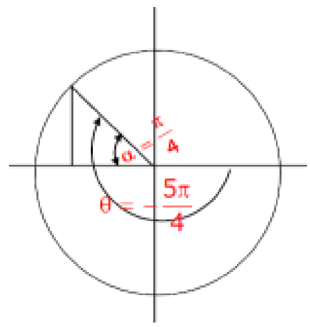
$x = 20(\sin(65°)) \approx 18$ ft.

The ladder will reach a point on the building approximately 18 feet above the ground.



13. What is the reference angle α for the angle $\theta = -\frac{5\pi}{4}$? Draw a diagram on the unit circle at the right to illustrate both θ and α and the reference triangle.

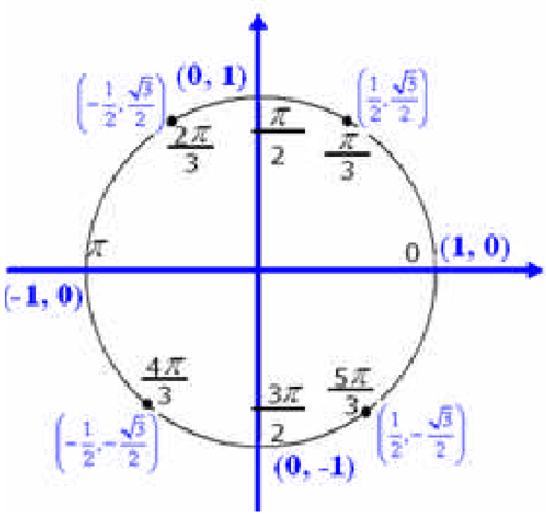
$$\alpha = \frac{\pi}{4}$$



14. Find the smallest positive θ for which $\sin(\theta) = -\frac{\sqrt{3}}{2}$

As seen in the unit circle at the right

$$\theta = \frac{4\pi}{3}$$



15. Consider the function f(x) = $\frac{1}{2}\cos\left(\frac{1}{3}x - \frac{\pi}{6}\right)$

What is the phase shift? $\frac{\pi}{2}$ What is the period? **6π**

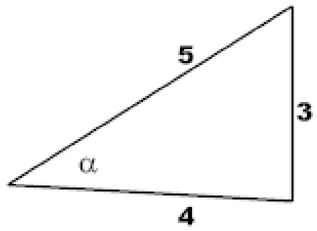
Show work here:

phase shift is $\frac{-C}{B} = \frac{\frac{\pi}{6}}{\frac{1}{3}} = \frac{\pi}{2}$

period is $\frac{2\pi}{B} = \frac{2\pi}{\frac{1}{3}} = 6\pi$

16. If tan(α) = $\frac{4}{3}$ and α is an acute angle, what is the exact value, expressed as a fraction, of cos(α)?

Hint: Draw the appropriate triangle. $\cos(\alpha) = \frac{4}{5}$

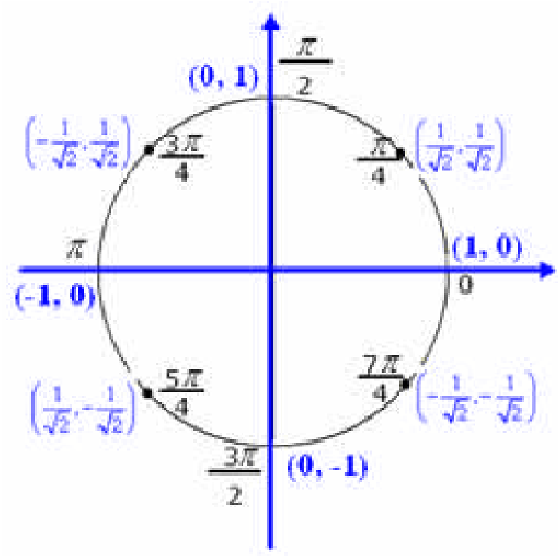


17. Find the exact value of $\beta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$.

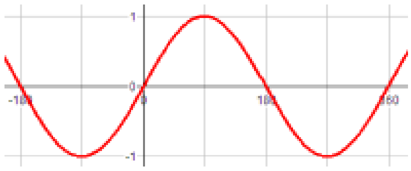
Use the unit circle at the right to answer this question and illustrate your process – locate the angle β. ~~Correlate β with one of the given angles.~~

Recall that $-\frac{\pi}{2} \le \beta \le \frac{\pi}{2}$

From the diagram at the right it is clear that $\beta = -\frac{\pi}{4}$



18. Sketch the graph of sin(x) from -180° to 360°



19. Solve the triangle at the right if $\alpha = 35.73°$ and b = 6.48
Your answers should be correct to two decimal places.

$\beta = 90^{\circ} - \beta = 90° - 35.73° = 54.27°$

$\cos(\alpha) = \frac{b}{c}$ from which we get $c = \frac{6.48}{\cos(35.73^{\circ})} \approx 7.98$

Or we could use the sin function as follows.

$\sin(\beta) = \frac{b}{c}$ from which we get $c = \frac{6.48}{\sin(54.27^{\circ})} \approx 7.98$

Now use the Pythagorean Theorem to obtain $a = \sqrt{c^2 - b^2} = \sqrt{7.98^2 - 6.48^2} \approx 4.66$

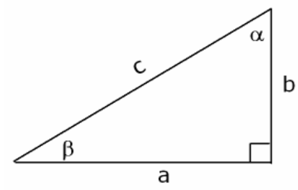
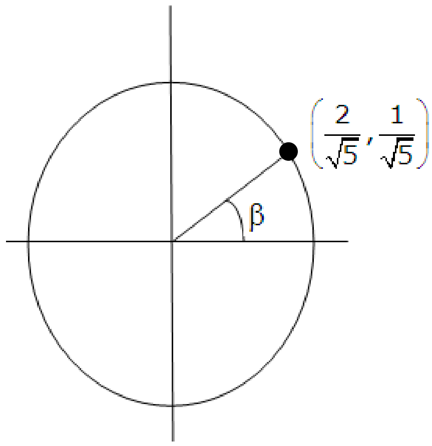


Figure 1

20. Refer to the diagram on the unit circle at the right. (Use a calculator)
Determine the measure of β in radians or degrees (your choice).
Your answer should be correct to two decimal places.
Label your answer as degree or radians.
Tell me the computations you did.



$\beta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 26.57°$ or $\beta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right) \approx 26.57°$

If you worked in radians you should have

$\beta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 0.464$ rad or $\beta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right) \approx 0.464$ rad

Converting 0.464 rad to degrees produces approximately 26.59° --- close enough!

21. What are (and why) the zeros of the function whose rule is $f(x) = \frac{x-5}{x+3}$

The zero of f is 5 because the zeros of a rational function are the zeros of the numerator which are not zeros of the denominator.

22. What are (and why) the vertical asymptotes for the function whose rule is $f(x) = \frac{3x^2 - 4x + 5}{2x^2 + 3x}$

The vertical asymptotes of a rational function occur at zeros of the denominator which are not zeros of the numerator.

Solve $0 = 2x^2 + 3x$ to obtain 0 and $\frac{-3}{2}$ as the zeros of the denominator.

The solutions of $3x^2 - 4x + 5 = 0$ are complex (because the discriminant is negative) .

It follows that the function f has vertical asymptotes $x = 0$ and a= $\frac{-3}{2}$

23. What is (and why) the horizontal asymptote for the function whose rule is $f(x) = \frac{3x^2 - 4x + 5}{2x^2 + 3x}$

Because the numerator and the denominator have the same degree, there is a horizontal asymptote and it is the line $y = \frac{3}{2}$

24. What is (and why) the slant asymptote for the function whose rule is $f(x) = \frac{3x^2 - 4x + 5}{x-1}$

Perform the indicated division

$$\begin{array}{r|l} & 3x - 1 \\ x - 1 & 3x^2 - 4x + 5 \\ & \underline{3x^2 - 3x} \\ & -x + 5 \\ & \underline{-x + 1} \\ & 4 \end{array}$$

Then write $f(x) = \frac{3x^2 - 4x + 5}{x-1} = (3x-1) + \frac{4}{x-1}$

From which we deduce that $g(x) = 3x - 1$ is the slant asymptote.

Because $\frac{4}{x-1} \longrightarrow 0$ as $x \longrightarrow \pm\infty$

25. What is (and why) the domain of the function whose rule is $f(x) = \frac{3x^2 - 4x + 5}{x-1}$

The domain of a rational function is all real numbers except the zeros of the denominator.

The doman of f is $\{x \mid x \neq 1\} = (-\infty, 1) \cup (1, \infty)$