SLU PreCalculus TEST 3 Sections 6.2 – 6.5, 7.1, 7.2, 11.1 – 11.4 Spring 2007

Course Average__________________

NAME: ______________________________________ Score______________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

You may use a calculator, but anytime you use the calculator your work must make it very clear to me what you did to arrive at your answer. <u>Exact means no decimal approximations.</u>

Each questions is worth 10 points.

1. Find the exact value of sin(75°). A calculator should not be used.

$\sin(75°)=\sin(30° + 45°) = \sin(30°)\cos(45°) + \sin(45°)\cos(30°)$

$$=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)+\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)=\frac{1+\sqrt{3}}{2\sqrt{2}}$$

2. The solutions for $2\cos(\theta)-\sqrt{3}=0$ in the interval $[0,2\pi]$ are 30° and 330°. Write an expression(s) for all solutions of $2\cos(\theta)-\sqrt{3}=0$.

$$\theta=\begin{cases}30^\circ+k360^\circ\\330^\circ+k360^\circ\end{cases}\text{ for } k\in\mathbb{Z}\qquad\text{or}\qquad\theta=\begin{cases}\dfrac{\pi}{6}+2k\pi\\[2mm]\dfrac{11\pi}{6}+2k\pi\end{cases}\text{ for } k\in\mathbb{Z}$$

3. Find **<u>ALL</u>** exact solutions in [0, 2π] for the equation $2\cos^2(x) - 3\sin(x) = 0$. A calculator should not be used.

$2\cos^2 x = 3\sin x = 0$

$2(1-\sin^2 x)+3\sin x = 0$

$2-2\sin^2 x + 3xinx = 0$

$2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$2\sin x + 1 = 0$ OR $\sin x - 2 = 0$

$\sin x = -\frac{1}{2}$ OR $\sin x = 2$

$x=\sin^{-1}(\sin(x))=\sin^{-1}\left(-\frac{1}{2}\right)=-30^\circ=\frac{11\pi}{6}$

and in Quadrant III $\frac{7\pi}{6}$ is also a solution

$x=\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right.$ OR $x\in\varnothing$ therefore $x=\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right.$ are the two solutions in the interval [0, 2π].

4. Find <u>ALL</u> solutions of the equation $\cos(2\theta)+\sin^2(\theta)=0$. A calculator should not be used.

$\cos(2\theta)+\sin^2(\theta)=0$

$\cos^2\theta-\sin^2\theta+\sin^2\theta=0$

$\cos^2\theta=0$

$\cos x = 0$

$$x=\begin{cases}\dfrac{\pi}{2}+2k\pi\\[2mm]\dfrac{3\pi}{2}+2k\pi\end{cases}\quad k\in\mathbb{Z}$$

$\cos(2\theta)+\sin^2(\theta)=0$

$1-2\sin^2\theta+\sin^2\theta=0$

$1-\sin^2\theta=0$

$\sin^2\theta=1$

$\sin\theta=\pm1$

$$x=\begin{cases}\dfrac{\pi}{2}+2k\pi\\[2mm]\dfrac{3\pi}{2}+2k\pi\end{cases}\quad k\in\mathbb{Z}$$

$\cos(2\theta)+\sin^2(\theta)=0$

$2\cos^2\theta-1+\sin^2\theta=0$

$2\cos^2\theta-(1-\sin^2\theta)=0$

$2\cos^2\theta-\cos^2\theta=0$

$\cos^2\theta=0$

$\cos\theta=0$

$$x=\begin{cases}\dfrac{\pi}{2}+2k\pi\\[2mm]\dfrac{3\pi}{2}+2k\pi\end{cases}\quad k\in\mathbb{Z}$$

5. Use a calculator to find a solution of 5.0118sin(x) – 3.1105 = 0 in Quadrant II. Show the work which tells me what computations you performed with the calculator.

$$5.0118\sin(x) - 3.1105 = 0$$

$$\sin x = \left(\frac{3.1105}{5.0118}\right)$$

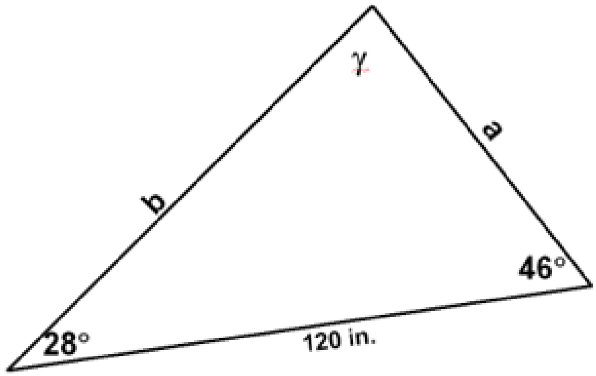
$$x = \sin^{-1}(\sin x) = \sin^{-1}\left(\frac{3.1105}{5.0118}\right) = 38.36^\circ \text{ in Quadrant I.}$$

The Quadrant II solution is $180^\circ - 38.36^\circ = 141.64^\circ$

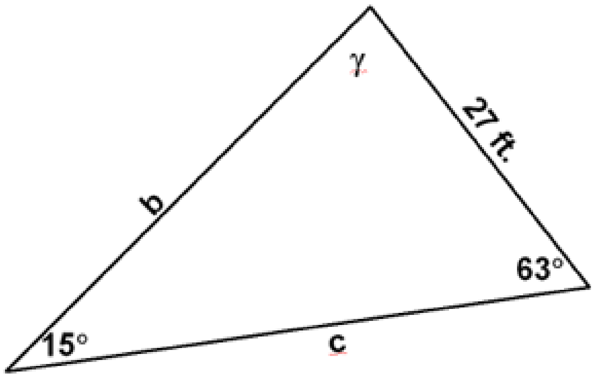
6. Find b in Triangle 1.

$$\gamma = 180 - 41 - 33 = 106^\circ$$

$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \Rightarrow b = \frac{c\sin\beta}{\sin\gamma} = \frac{(21)\sin 33^\circ}{\sin 106^\circ} = 11.9$$



Triangle 1



Triangle 2

7. Find b in Triangle 2.

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$b = \sqrt{13.7^2 + 20.1^2 - (2)(13.7)(20.1)\cos 27.3} = 10.11$$

8. Prove $\tan^2(x) - \sin^2(x) = \tan^2(x)\sin^2(x)$

$$\tan^2(x) - \sin^2(x) = \frac{\sin^2 x}{\cos^2 x} - \sin^2 x = \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x\sin^2 x}{\cos^2 x} = \frac{\sin^2 x - \cos^2 x\sin^2 x}{\cos^2 x}$$

$$\frac{\sin^2 x(1-\cos^2 x)}{\cos^2 x} = \left(\frac{\sin^2 x}{\cos^2 x}\right)\left(1-\cos^2 x\right) = \tan^2 x\sin^2 x$$

9. Prove $\sin(2x) = (\tan(x))(1+\cos(2x))$

$$\tan x\,(1+\cos(2x)) = \tan\left(1+\cos^2 x - \sin^2 x\right) = \left(\frac{\sin x}{\cos x}\right)\left(\cos^2 x + (1-\sin^2 x)\right) = \left(\frac{\sin x}{\cos x}\right)\left(\cos^2 x + \cos^2 x\right)$$

$$= \left(\frac{\sin x}{\cos x}\right)\left(2\cos^2 x\right) = 2\sin x\cos x = \sin(2x)$$

$$\tan x\,(1+\cos(2x)) = \left(\frac{\sin x}{\cos x}\right)\left(1+2\cos^2 x - 1\right) = \left(\frac{\sin x}{\cos x}\right)\left(2\cos^2 x\right) = 2\sin x\cos x = \sin(2x)$$

$$\tan x\,(1+\cos(2x)) = \left(\frac{\sin x}{\cos x}\right)\left(1+1-2\sin^2 x\right) = \left(\frac{\sin x}{\cos x}\right)(2)\left(1-\sin^2 x\right) = \left(\frac{\sin x}{\cos x}\right)(2)\left(\cos^2 x\right) = 2\sin x\cos x = \sin(2x)$$

10. Consider $\frac{(x-2)^2}{12} + \frac{(y+3)^2}{8} = 1$

To analyze this equation it is recommended that it be translated to a new u-v coordinate system.
Fill in the blanks and answer the following questions to complete the analysis.

a) let u = x-2 and v = y+3

If u = 0, then x = 2 and if v = 0, then y = -3

This implies the origin of the new translated u-v system has coordinates (2, -3) in the original x-y coordinate system.

b) Sketch the new translated u-v coordinate system on the original coordinate system in Figure 1.

c) Write the equation of the curve in the translated system $\frac{u^2}{12} + \frac{v^2}{8} = 1$

d) If u = 0, then v = $\pm\sqrt{8}$ which implies the v-intercepts are (0, $\pm\sqrt{8}$).

If v = 0, then u = $\pm\sqrt{12}$ which implies the u-intercepts are ($\pm\sqrt{12}$,0)

e) Sketch the graph (on the translated coordinate system) and label its vertices with coordinates in the translated coordinate system.