SLU PreCalculus Quiz 11 Solution Fall 2008

Name _____ Score_____/10

Please Print Clearly
Show your work neatly and well organized.

1. 1) To find the all solutions of $5\sin(\theta) - 3 = 0$ in the interval $[0, 2\pi]$ we would write

$$\sin(\theta) = \frac{3}{5}.$$

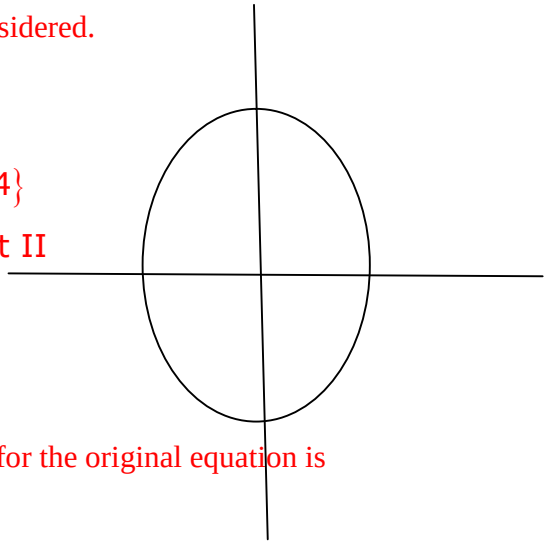
Then we would use the inverse sin function to obtain $\theta = \sin^{-1}(\sin(\theta)) = \sin^{-1}\left(\frac{3}{5}\right) \approx 36.87^\circ$.

Complete the analysis to find all solutions in the interval $[0, 2\pi]$. Make sure that you justify your answer.

In the interval $[0, 2\pi]$ there are four possibilities that must be considered.

$\theta_1 = 36.87$ $\theta_2 = 180 - \theta_1 = 143.13$

$\theta_3 = 180 + \theta_1 = 216.87$ $\phi_4 = 360 - \theta_1 = 323.13$



We consider these four because $\left|\sin(\theta_i)\right| = \frac{3}{5}$ for $i \in \{1, 2, 3, 4\}$

$\sin\alpha > 0$ if and only if α is in Quadrant I or Quadrant II

$$\sin(36.87) = \sin(143.13) = \frac{3}{5} \text{ and}$$

Therefore

$$\sin(216.87) = \sin(323.13) = -\frac{3}{5}$$

From which it follows that the solution set in the interval $[0, 2\pi]$ for the original equation is $\{36.87^\circ, 143.13^\circ\}$

2) Find all **EXACT** solution of $\sin^2(x) - \sin(x)\cos(x) = 0$ in the interval $[0, 2\pi]$.

$\sin^2(x) - \sin(x)\cos(x) = 0 \Rightarrow \sin(x)(\sin(x) - \cos(x)) = 0$

$\Rightarrow \sin(x) = 0$ OR $\sin(x) - \cos(x) = 0$

In the interval $[0, 2\pi]$, $\sin(x) = 0$ has the solutions 0, π, and 2π.

To find all solutions of $\sin(x) - \cos(x) = 0$ in the interval $[0, 2\pi]$, we observe that in the first quadrant $\sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$. Therefore $x_1 = \frac{\pi}{4}$ is a solution of $\sin(x) - \cos(x) = 0$.

Other possibilities in $[0, 2\pi]$ are

$x_2 = \frac{3\pi}{4}$ in Quadrant II, $x_3 = \frac{5\pi}{4}$ in Quadrant III, and $x_4 = \frac{7\pi}{4}$ in Quadrant IV

For each of these $\left|\sin(x_i)\right| = \left|\cos(x_i)\right|$, but sine and cosine have the same sign only in Quadrants I and III. Therefore

$\frac{\pi}{4}$ and $\frac{5\pi}{4}$ are the only solutions of $\sin(x) - \cos(x) = 0$ in $[0, 2\pi]$.

The solution set in $[0, 2\pi]$ for the original equation is $\left\{0, \pi, 2\pi, \frac{\pi}{4}, \frac{5\pi}{4}\right\}$