

NAME: _____ Score _____/10

Please **print** your name

You must show your work in order to receive credit.

Each of the following questions refer to the function f whose rule is

$$f(x) = \frac{(x+2)(x-3)(x+3)(x+4)}{2x(x-3)(x+4)(x+1)}$$

- f is a **rational** function.
- What is the domain of f ? The domain of f is all real numbers except the zeros of the denominator. Clearly the zeros of the denominator are 0, 3, -4, and -1. Therefore the domain of f is all real numbers except 0, 3, -1, -4.
Written in set builder notation $D_f = \{x \in \mathbb{R} \mid x \neq -4, -1, 0, 3\}$.
Written in interval notation $D_f = (-\infty, -4) \cup (-4, -1) \cup (-1, 0) \cup (0, 3) \cup (3, \infty)$
- What are the zeros of f ? The zeros of f are the zeros of the numerator which are not zeros of the denominator.
Zeros of the denominator are easily seen to be -4, -1, 0, 3
Zeros of the numerator are easily seen to be -2, 3, -3, -4
Zeros of f are therefore -2 and -3.
- What are the vertical asymptotes of the graph of f . Vertical asymptotes occur at zeros of the denominator which are not zeros of the numerator. Therefore the vertical asymptotes occur at 0 and -1.
Because vertical asymptotes are vertical lines, the vertical asymptotes are the lines $x = 0$ and $x = -1$.
- If the graph of f has a horizontal asymptote what is it?
The numerator of f is a fourth degree polynomial whose leading coefficient is 1.
The denominator of f is a fourth degree polynomial whose leading coefficient is 2.
Because the numerator and the denominator have the same degree the function f has a horizontal asymptote and it is the line $y = \frac{1}{2}$

REMARKS:

- The domain of a function is a set and should be described as a set.
- A zero of a function is a number in the domain and should be described as a number. This can be done with a simple list or the set of all zeros can be described with common accepted set notation.
- An asymptote is a line and should be described as a line not as a number.
- When we ask that you show your work in this and higher level math classes, we do not expect to see minor arithmetic details. Rather than minor arithmetic we expect to see the statements of reasons, principles, and logic used to arrive at the conclusion. The solutions presented above provide a good example.