SLU PreCalculus Quiz 8 Fall 2008

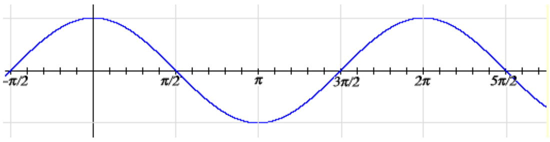
The following may be helpful

$\sin(0) = 0 \quad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \quad \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \sin\left(\frac{\pi}{2}\right) = 1$

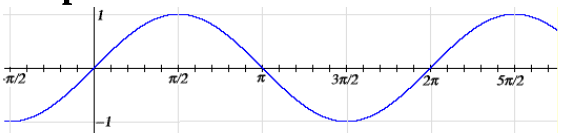
$\cos(0) = 1 \quad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \quad \cos\left(\frac{\pi}{2}\right) = 0$

cos (x – y) = cos(x)cos(y) + sin(x)sin(y) sin(x – y) = sin(x)cos(y) – cos(x)sin(y)
cos (x + y) = cos(x)cos(y) – sin(x)sin(y) sin(x + y) = sin(x)cos(y) + cos(x)sin(y)
sin(2x) = 2sin(x)cos(x)
$\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$

Graph of The Cosine Function:



Graph of The Sine Function:



1. Without a calculator. Calculate the exact value of sin(105°). Show your work neatly.

$\sin(105^\circ) = \sin\left(60^\circ + 45^\circ\right) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}$

Alternate (not desirable):

$\sin(105^\circ) = \sin\left(150^\circ - 45^\circ\right) = \sin 150^\circ \cos 45^\circ - \cos 150^\circ \sin 45^\circ$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

Alternate (not desirable):

$\sin(105^\circ) = \sin\left(\frac{210^\circ}{2}\right) = \sqrt{\frac{1-\cos 210}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$

This hardly looks like the earlier answers, but a check with a calculator shows both are about 0.9659 and of course we know the results must be equal. How do you show they are equal? Here is one way.

$\sqrt{\frac{2+\sqrt{3}}{4}} = \sqrt{\frac{4+2\sqrt{3}}{4(2)}} = \sqrt{\frac{1+2\sqrt{3}+3}{4(2)}} = \sqrt{\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)^2} = \frac{1+\sqrt{3}}{2\sqrt{2}}$

2. Based on the graph shown at the right, how many solutions are there to the equation
sin(x) = 0.004x – 0.25 ?

There are three solutions. They are the first coordinates of the three points of intersection of the two curves.

Read the next question carefully!

Mark the solutions for the above equation with a big, heavy, easy to see dots.Make your dots about this size •