SLU PreCalculus TEST 1 Fall 2008

NAME: ______________________________ Score______________________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Questions 1 – 15 are each worth 2 points. Questions 16 – 25 are each worth 5 points. The last three questions combined are worth 20 points.
Circle T or F, whichever is correct.

1. T F Every function is a polynomial function.

2. T F To verify (prove) that two functions f and f^{-1} are inverses of each other it is necessary to show that both of the following are true:

   $f^{-1} \circ f(x) = x$ for all x in the domain of f and

   $f \circ f^{-1}(x) = x$ for all x in the domain of f^{-1}

3. T F f(x) is the rule for the function f.

4. T F If the multiplicity of a real zero is even the graph of the function crosses the x-axis at that zero.

5. T F If the graph of a function passes the horizontal line test, the function has an inverse.

6. T F The product of two functions is a real number.

Fill in each of the blanks to make the statements true.

7. The graph of a function is the set of all points whose coordinates are (a, f(a)) where a is an element of the domain.

8. If p and d are polynomials with real coefficients, then there are unique polynomials q and r with real coefficients such that p = dq + r with r = 0 or the degree of r is less than the degree of d.

9. The composition of a function f with a function g is a function named $f \circ g$ whose rule is $f \circ g(x) = f\left(g(x)\right)$

10. If f is a polynomial function such that f(a) < 0 and f(b) > 0, then f has an x-intercept between a and b.

11. If a horizontal line may be drawn so that it intersects the graph of a function in more than one point, then the function does not have an inverse.

12. If $\frac{p}{q}$ is a rational zero of a polynomial function with integer coefficients, then the numerator p must be a divisor of the constant term and the denominator q must be a divisor of the leading coefficient.

13. The degree of the function f whose rule is $f(x) = 3x^5 - 7x^3 + 2x + 7$ is 5

14. The graph of the function whose rule is $f(x) = -3x^2 + 2x - 98$ is a parabola which opens down.

15. The slope of the line through the points (–3, 2) and (7, –5) is $-\frac{7}{10}$

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2+5}{-3-7} = \frac{-7}{10}$$

For Problems 16 – 25, Show all necessary work. NO WORK – NO CREDIT
Sentences are good

16. What is the domain of the function whose rule is $f(x) = \dfrac{3x+4}{x-4}$?

By convention the domain is the largest set of real numbers for which the rule makes sense. The domain of f is therefore the set of all real numbers except 4. Because the denominator cannot be zero.

Could be written as $(-\infty, 4) \cup (4, \infty)$ or as $\{x \mid x \neq 4\}$

17. The rule for a function f is given by the equation $f(x) = \dfrac{x-3}{2x+1}$ and the rule for a function g is given by the equation g(x) = x – 2. Determine the rule for the function $f \circ g$.

$$f \circ g(x) = f(g(x)) = f(x-2) = \frac{(x-2)-3}{2(x-2)+1} = \frac{x-5}{2x-3}.$$

When the rule for a function is written using function notation in an equation, one side (usually the left) of the equation **must be** the range element and the other side **must be** the recipe. Consequently your work should begin with $f \circ g(x)$. Remember $f \circ g$ is the name of the function, so it makes no sense what-so-ever to write $f \circ g = \dfrac{x-5}{2x-3}$. Notice in the above answer to the question I started with the range element followed by what the definition tells me, continuing until the expression is simplified.

Note there is no need to write another statement like $f \circ g(x) = \dfrac{x-5}{2x-3}$ because that is already contained in the string of equalities. A property of equality is: **If a = b and b = c, then a = c.**

18. Suppose f and g are functions whose rules are $f(x) = x^2 - 3$ and $g(x) = \dfrac{x}{3}$. Calculate $f \circ g(2)$.

This calculation should start with $f \circ g(2)$.

$$f \circ g(2) = f(g(2)) = f\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^2 - 3 = \frac{4}{9} - 3 = \frac{-23}{9}$$

19. Find the inverse of the function whose rule is $f(x) = 5x + 3$.

$f(x) = 5x + 3$

$y = 5x + 3$

$x = 5y + 3$

$y = \dfrac{x-3}{5}$

$f^{-1}(x) = \dfrac{x-3}{5}$

20. Suppose f and g are functions whose rules are f(x) = 3x + 2 and $g(x) = \dfrac{1}{3}x - 2$.

Verify (prove) that f and g are **not** inverses of each other.

$$f \circ g(x) = f(g(x)) = f\left(\frac{1}{3}x - 2\right) = 3\left(\frac{1}{3}x - 2\right) + 2 = x - 4 \neq x$$

Therefore f and g are not inverses of each other.

21. The rule for a function f is $f(x) = -3x^5 - 22x^4 + 6x^3 - 7x^2 + 8x - 5$ .Complete the following statements about f.

   a. The graph of f "tries" to cross the x-axis 5 times.

   b. The graph of f can cross the x-axis no more than 5 times.

   c. The graph of f must cross the x-axis at least 1 times.

   d. $\lim_{x\to\infty} f(x) = -\infty$

      $\lim_{x\to-\infty} f(x) = \infty$

22. Perform the polynomial division. $x^2 - 2\overline{)x^3 - 4x^2 - 7x + 1}$

$$\begin{array}{r} x - 4 \\ x^2 - 2\overline{)x^3 - 4x^2 - 7x + 1} \\ \underline{x^3 \quad\;\; - 2x \quad\;\;} \\ -4x^2 - 5x + 1 \\ \underline{-4x^2 \quad\;\; + 8} \\ -5x - 7 \end{array}$$

23. What are the possible rational zeros of the function whose rule is $f(x) = 5x^3 + 2x^2 - 7x - 7$?

$p \in \{\pm 1, \pm 7\}$

$q \in \{\pm 1, \pm 5\}$

$\frac{p}{q} \in \left\{\pm 1, \pm 7, \pm \frac{1}{5}, \pm \frac{7}{5}\right\}$

24. An analysis of a function f reveals the following facts.

   a. f is a polynomial function of degree 4.

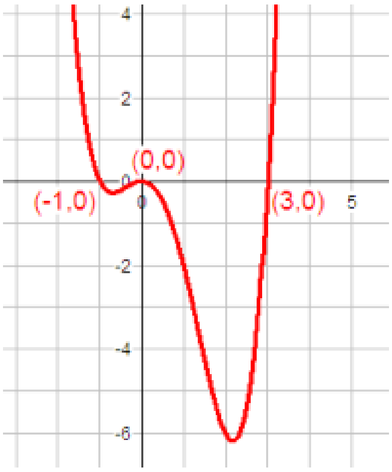
   b. As $x \longrightarrow -\infty$, $f(x) \longrightarrow +\infty$

      As $x \longrightarrow +\infty$, $f(x) \longrightarrow +\infty$

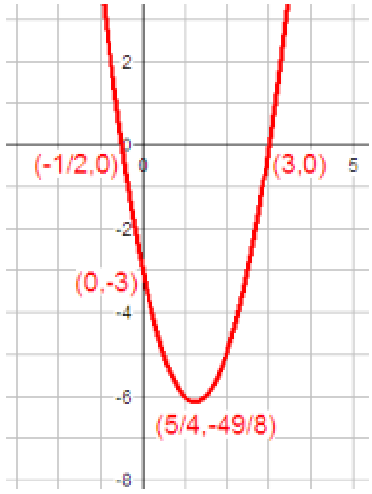
   c. The real zeros of f are -1, 0, and 3.

   d. The multiplicity of 0 is 2.

Sketch the graph of f.



25. Sketch the graph of the function whose rule is $f(x) = 2x^2 - 5x - 3$
Label the x and y intercepts and the vertex with their coordinates. Factoring helps. Label important points.



$f(x) = 2x^2 - 5x - 3 = (2x+1)(x-3)$

So f(x) = 0 if and only if x = 3 or x = -1/2

The x-intercepts are (3, 0) and (-1/2, 0)

The vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(\frac{5}{4}, f\left(\frac{5}{4}\right)\right) = \left(\frac{5}{4}, -\frac{49}{8}\right)$

$$f\left(\frac{5}{4}\right) = \left(2\left(\frac{5}{4}\right)+1\right)\left(\frac{5}{4}-3\right) = \left(\frac{5}{2}+1\right)\left(\frac{5}{4}-\frac{12}{4}\right)$$

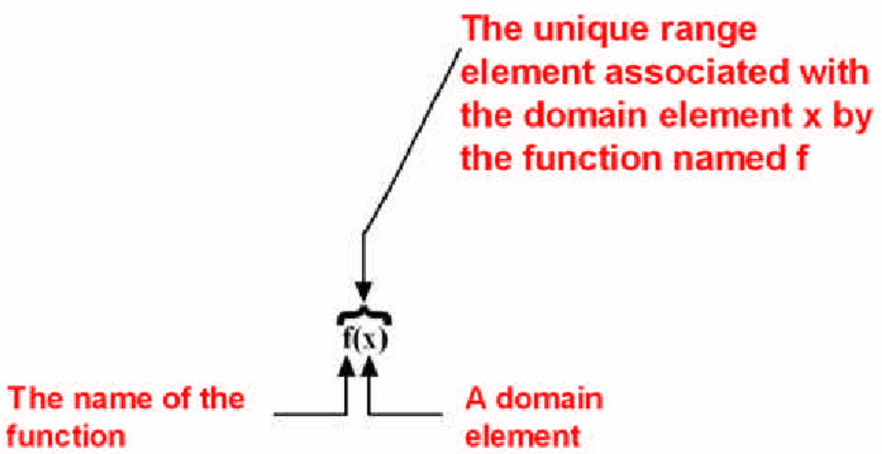
$$= \left(\frac{7}{2}\right)\left(-\frac{7}{4}\right) = -\frac{49}{8}$$

26. (6 points) Fill in the blanks

Definition: A **function** consists of three things

- A set called the **domain**
- A set called the **range**
- A **rule** which associates **each** element of the domain with a **unique** element of the range.

27. (4 points)



28. (10 points) Rules for functions are given at the top of the page and graphs of functions are given below them. Match the graphs and the rules by writing the letter which identifies a graph in the blank preceding a rule for a function.

a. __A__ $f(x) = \frac{1}{x}$ b. __E__ $f(x) = -3x + 3$

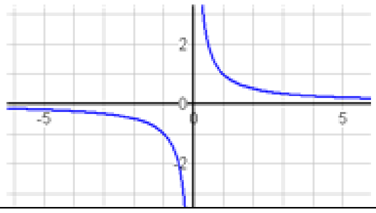
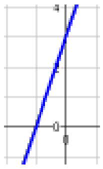
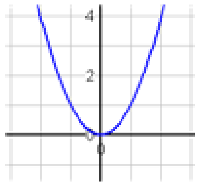
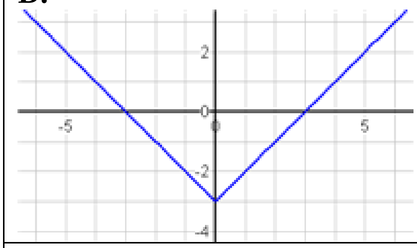
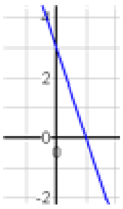
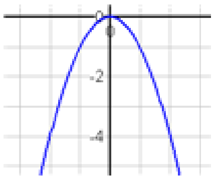
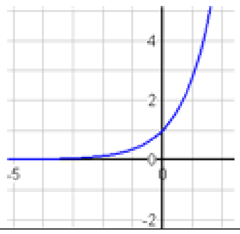
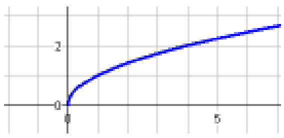
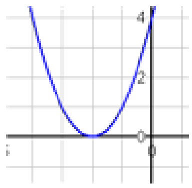
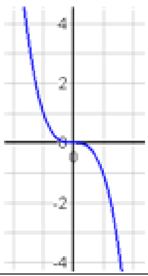
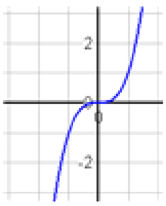
c. __F__ $f(x) = -x^2$ d. __B__ $f(x) = 3x + 3$

e. __C__ $f(x) = x^2$ f. __I__ $f(x) = (x + 2)^2$

g. __D__ $f(x) = |x| - 3$ h. __H__ $f(x) = \sqrt{x}$

k. __K__ $f(x) = x^3$ m. __J__ $f(x) = -x^3$

The graphs are shown in blue.

A. 	**B.** 	**C.** 
D. 	**E.** 	**F.** 
G. 	**H.** 	**I.** 
J. 	**K.** 	**L.**