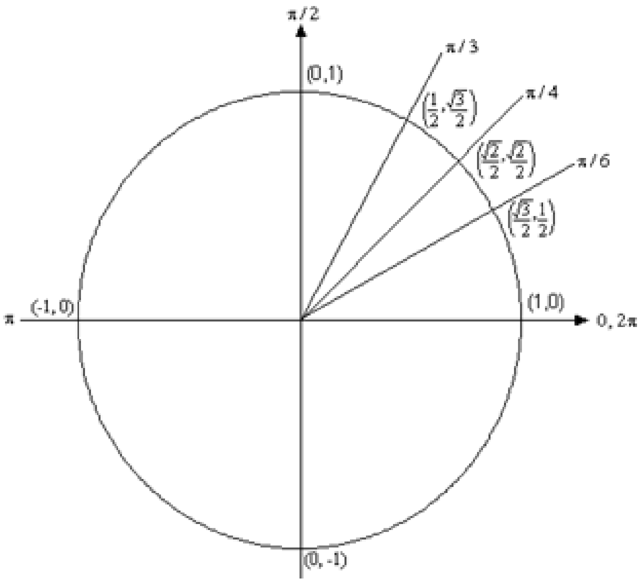
SLU PreCalculus TEST 2 Solution Fall 2008

NAME: ______________________________ Score____________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

Questions are each worth 4 points.



1. $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

2. What is the exact value of cos(225°)? $-\frac{\sqrt{2}}{2}$

3. The range of the cos function is [-1, 1]

4. The domain of the ln function is (0, ∞)

5. Circle those values in the following list which are zeros of the sin function.

   $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}$

6. Circle those values in the following list which are zeros of the cos function.

   $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, 2\pi, \frac{5\pi}{2}$

7. If cos(θ) < 0 and tan(θ) > 0, then θ is in Quadrant **III**.

8. Find the exact value of $\ln\left(e^{\sqrt{7}}\right)$ $\sqrt{7}$

9. What is the domain of the function whose rule is $f(x) = \frac{3x^2 - 4x + 5}{2x^2 + 3x}$?

The domain of f is all real numbers except real zeros of $2x^2 + 3x = x(2x + 3)$

Therefore the domain of f is all real except 0 and $-\frac{3}{2}$

Alternate ways of writing this are:

$D_f = \left\{x \in R \,\middle|\, x \neq 0 \text{ and } x \neq -\frac{3}{2}\right\}$ or $D_f = \left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, 0\right) \cup (0, \infty)$

10. In a discussion of the $\sin^{-1}$ function the domain of sin function is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

11. In a right triangle with acute angles $\alpha = \frac{2\pi}{5}$ and β, the exact value of β is $\frac{\pi}{10}$

$$\beta = \frac{\pi}{2} - \frac{2\pi}{5} = \frac{5\pi}{10} - \frac{4\pi}{10} = \frac{\pi}{10}$$

12. What is the amplitude of the 3sin(5x + $\frac{\pi}{3}$) ? 3   amplitude is A

For Questions 12 – 14, compare to f(x) = Asin(Bx+C)

13. What is the period of the 3sin(5x + $\frac{\pi}{3}$) ? $\frac{2\pi}{B} = \frac{2\pi}{5}$

14. What is the phase shift of the 3sin(5x + $\frac{\pi}{3}$) ? $-\frac{C}{B} = -\frac{\frac{\pi}{3}}{5} = -\frac{\pi}{15}$

For Problems 15 – 25,   Show all necessary work.   NO WORK – NO CREDIT
Sentences are good. Lines of equalities are very good. If two expressions are equal say that by using the = symbol. If two expressions are not equal do not put the = symbol between them.
If two expressions are approximately equal say that by using the symbol ≈.

15. In Figure 1 at the right, b = 9, a = 40, and c = 41. Calculate sin(β).
Write your answer as a fraction (<u>no decimals</u>).

$$\sin\beta = \frac{opp}{hyp} = \frac{b}{c} = \frac{9}{41}$$

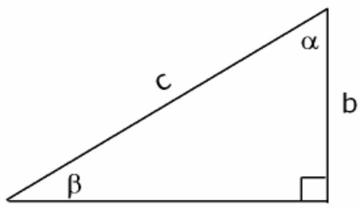


Figure 1

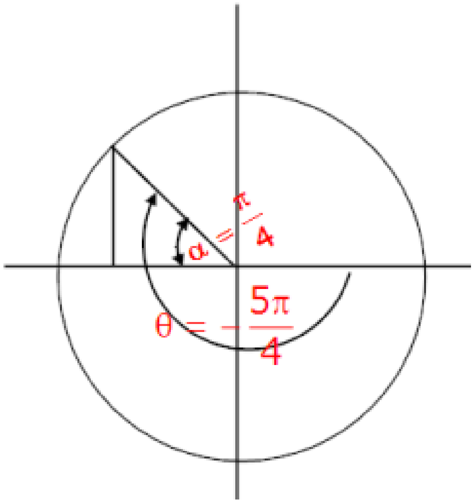
16. Find the <u>exact</u> solutions of the equation $e^{3x+2} = 7$.

$$\ln\left(e^{3x+2}\right) = \ln(7)$$
$$3x + 2 = \ln(7)$$
$$3x = -2 + \ln(7)$$
$$x = \frac{-2 + \ln(7)}{3}$$

17. What is the reference angle α for the angle $\theta = -\frac{5\pi}{4}$?

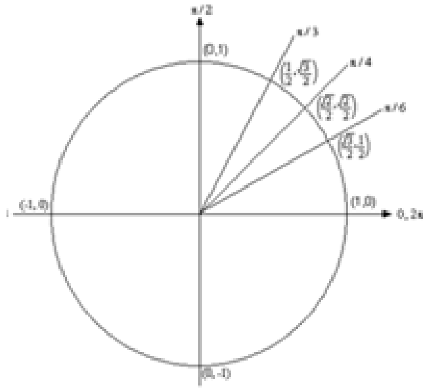
Draw a diagram on the unit circle at the right to illustrate both θ and α and the reference triangle. Write the value of α.

$\alpha = \frac{\pi}{4}$



18. Find the smallest positive θ for which $\sin(\theta) = -\frac{\sqrt{3}}{2}$

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3} = 240^\circ$



19. If $\tan(\alpha) = \frac{5}{12}$ and α is an acute angle, what is the exact value, expressed as a fraction, of $\sin(\alpha)$? Hint: Draw the appropriate triangle.

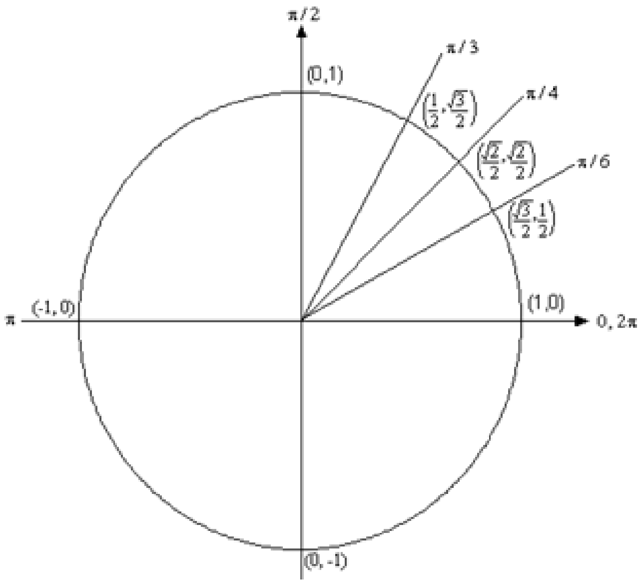
$\tan\alpha = \frac{opp}{adj} = \frac{5}{12}$

$\sin\alpha = \frac{opp}{hyp} = \frac{5}{13}$

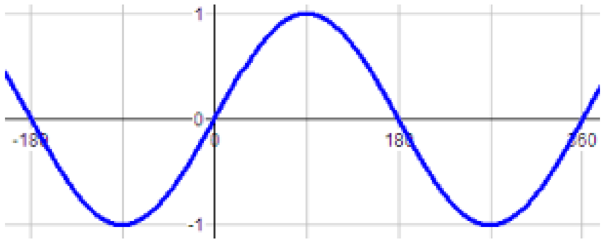
20. Find the exact value of $\beta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

Use the unit circle at the right to answer this question and illustrate your process – locate the angle β. Draw and label the angle β on the unit circle at the right.

$$\beta = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) \Rightarrow \left\{\begin{array}{l} -\frac{\pi}{2} \le \beta \le \frac{\pi}{2} \\ \sin\beta = -\frac{\sqrt{3}}{3} \end{array}\right\} \Rightarrow \beta = \frac{-\pi}{3}$$



21. Sketch the graph of sin(x) from -180° to 360°



22. Solve the triangle at the right if α = 35.73° and b = 6.48
Your answers should be correct to two decimal places. Your work should be organized in a neat and readable fashion.

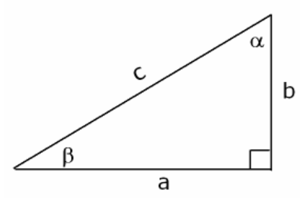


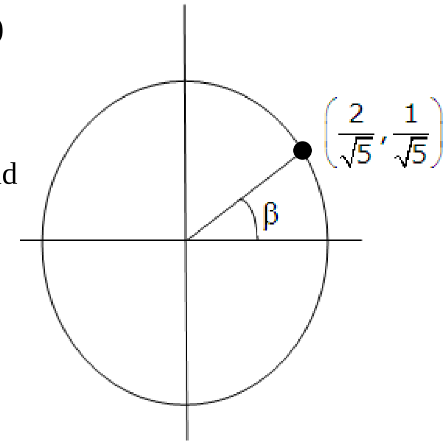
Figure 1

$$\beta = 90° - \alpha = 90° - 35.73° = 54.27°$$

$$\sin\beta = \frac{b}{c} \Rightarrow c = \frac{b}{\sin\beta} = \frac{6.48}{\sin 54.27°} = 7.98$$

$$\sin\alpha = \frac{a}{c} \Rightarrow a = c\sin\alpha = 7.98\sin 35.73° = 4.66$$

23. Refer to the diagram on the unit circle at the right. (Use a calculator)
Determine the measure of β in radians or degrees (your choice).
Your answer should be correct to two decimal places.
Label your answer as degree or radians.
Tell me (preferably with a line of equalities) the computations you did



$\beta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 26.57°$

or

$\beta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right) \approx 26.57°$

or

$\beta = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 0.46$ radians

or

$\beta = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right) \approx 0.46$ radians

24. What are (and why) the vertical asymptotes for the function whose rule is

$$f(x) = \frac{(x-2)(x+3)(x-4)}{(x+3)(x-5)}$$

Vertical asymptotes occur at zeros of the denominator which are not zeros of the numerator.
Therefore f has a vertical asymptote x = 5.

25. What is (and why) the horizontal asymptote for the function whose rule is $f(x) = \frac{5x^3 - 4x^2 + 5}{3x^3 + 3x}$

Because the numerator and denominator have the same degree, the function has a horizontal asymptote.
The y-intercept of the horizontal asymptote is the quotient of the leading coefficients.
The horizontal asymptote is $y = \frac{5}{3}$