SLU PreCalculus TEST 3 Fall 2008

NAME: ______________________________ Score______________________/100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

The last page of this test contains formulas and facts which may be helpful. Detach it and use it.

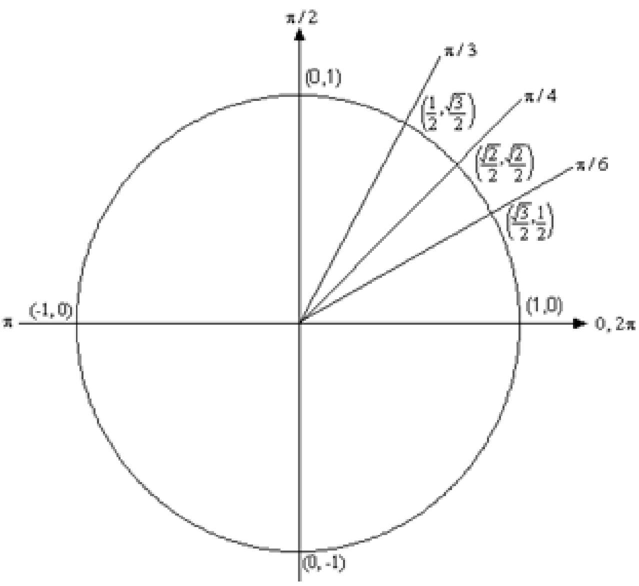
Questions 1 - 10 are each worth 4 points. Questions 11 – 16 are each worth 10 points.

1. Find the **exact** value of sin(75°). A calculator should not be used.

$\sin(75) = \sin(30+45)$

$= \sin(30)\cos(45) + \cos(30)\sin(45)$

$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$



2. What is the **exact** value of cos(330°)? $\frac{\sqrt{3}}{2}$

3. $(x-h)^2 + (y-k)^2 = 5$ is the equation of a **circle** with center at **(h, k)**

4. $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{a^2} = 1$ is the equation of a **hyperbola** with center at **(h, k)**

5. $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{a^2} = 1$ is the equation of a **ellipse** with center at **(h, k)**

6. In a discussion of the $\sin^{-1}$ function the domain of sin function is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

7. In a discussion of the $\cos^{-1}$ function the domain of cos function is restricted to the interval **[0, π]**

8. What is the solution set for the equation sin(x) = –2.**The null set ∅**

9. A particular trigonometric equation involving only the sin function has solutions $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ in the interval [0, 2π]. Write expression(s) for ALL real solutions of the equation.

$\frac{\pi}{3} + 2k\pi$ for $k \in \mathbb{Z}$ and $\frac{5\pi}{3} + 2k\pi$ for $k \in \mathbb{Z}$

10. Provide a counterexample to show that sin(x) – cos(x) = 1 is NOT an identity.

Let x = 0. Then sin(x) – cos(x) = 0 – 1 = –1 ≠ 1

11. To find the all solutions of $5\sin(\theta) - 3 = 0$ in the interval $[0, 2\pi]$ we would write $\sin(\theta) = \frac{3}{5}$.

Then we would use the inverse sin function to obtain $\theta = \sin^{-1}(\sin(\theta)) = \sin^{-1}\left(\frac{3}{5}\right) \approx 36.87^\circ$.

Complete the analysis to find all solutions in the interval $[0, 2\pi]$

In the interval $[0, 2\pi]$ there are four possibilities that must be considered.

$\theta_1 = 36.87$ $\theta_2 = 180 - \theta_1 = 143.13$

$\theta_3 = 180 + \theta_1 = 216.87$ $\phi_4 = 360 - \theta_1 = 323.13$

We consider these four because $|\sin(\theta_i)| = \frac{3}{5}$ for $i \in \{1,2,3,4\}$

$\sin\alpha > 0$ if and only if α is in Quadrant I or Quadrant II

Therefore

$$\sin(36.87) = \sin(143.13) = \frac{3}{5} \text{ and}$$

$$\sin(216.87) = \sin(323.13) = -\frac{3}{5}$$

From which it follows that the solution set in the interval $[0, 2\pi]$ for the original equation is

$\{36.87^\circ, 143.13^\circ\}$

12. Prove the identity $(\sec(x) - \tan(x))^2 = \frac{1-\sin(x)}{1+\sin(x)}$

Proof:

$$\frac{1-\sin x}{1+\sin x} = \frac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)} = \frac{(1-\sin x)^2}{1-\sin^2 x} = \frac{(1-\sin x)^2}{\cos^2 x} = \left(\frac{1-\sin x}{\cos x}\right)^2$$

$$= \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2 = (\sec x - \tan x)^2$$

13. Find the **<u>exact</u>** value of $\cos(74^\circ)\cos(44^\circ) + \sin(74^\circ)\sin(44^\circ)$. Do not use your calculator.

$$\cos(74^\circ)\cos(44^\circ) + \sin(74^\circ)\sin(44^\circ) = \cos(74^\circ - 44^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

14. In a triangle with the conventional labeling that has been used in class and text,
$a = 10.3$, $c = 6.45$, and $\beta = 32.4^\circ$. **Calculate b.**

Use the Law of Cosines

$b^2 = a^2 + c^2 - 2ac\cos\beta$ or equivalently $b = \sqrt{a^2 + c^2 - 2ac\cos\beta}$

from which it follows that

$$b = \sqrt{10.3^2 + 6.45^2 - 2(10.3)(6.45)\cos(32.4^\circ)} = \sqrt{35.5} = 5.96$$

15. Solve the triangle (s) with $a = 27$, $b = 62$, and $\alpha = 81^\circ$. (Be thoughtful)

Use the Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} \Rightarrow \sin\beta = \frac{b\sin\beta}{a}$$

$$\Rightarrow \sin\beta = \frac{62\sin 81^\circ}{27} = 2.268 > 1$$

There is no such angle β.

Therefore there is no triangle.

16. Sketch the graph of $\frac{x^2}{3} + \frac{y^2}{4} = 2$. Show and label (with coordinates) all vertices, Foci and any other important points

$$\frac{x^2}{3} + \frac{y^2}{4} = 2 \Leftrightarrow \frac{x^2}{6} + \frac{y^2}{8} = 1$$

The graph is an ellipse.

$x = 0 \Rightarrow y^2 = 8 \Rightarrow y = \pm\sqrt{8}$

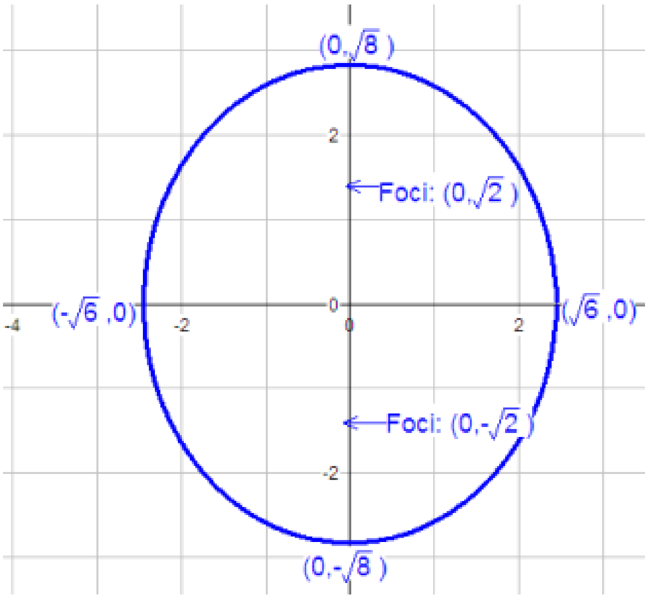
Plot the points $(0, \sqrt{8})$ and $(0, -\sqrt{8})$

$y = 0 \Rightarrow x^2 = 6 \Rightarrow x = \pm\sqrt{6}$

Plot the points $(0, \sqrt{6})$ and $(0, -\sqrt{6})$

$c^2 = 8 - 6 = 2 \Rightarrow c = \pm\sqrt{2}$

Foci are: $(0, \sqrt{2})$ and $(0, -\sqrt{2})$



An ellipse
Major axis is the y-axis
Minor axis is the x axis