

Name _____ Score _____/10

Please Print Clearly1. Prove the identity $\cos^4\beta - \sin^4\beta = \cos(2\beta)$ **Proof:**

$$\cos^4\beta - \sin^4\beta = (\cos^2\beta - \sin^2\beta)(\cos^2\beta + \sin^2\beta) = (\cos^2\beta - \sin^2\beta)(1) = \cos(2\beta)$$

2. Use the identity $\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$ to prove the identity $\cos(180^\circ - \beta) = -\cos(\beta)$ **Proof:**

$$\cos(180^\circ - \beta) = \cos(180)\cos(\beta) + \sin(180)\sin(\beta) = (-1)\cos(\beta) + (0)\sin(\beta) = -\cos(\beta)$$

3. Prove the identity $\cos(\alpha + 60^\circ) + \sin(30^\circ + \alpha) = \cos(\alpha)$ **Proof:**

$$\begin{aligned} \cos(\alpha + 60^\circ) + \sin(30^\circ + \alpha) &= \cos(\alpha)\cos(60) - \sin(\alpha)\sin(60) + \cos(30)\sin(\alpha) + \cos(\alpha)\sin(30) \\ &= \cos(\alpha)\left(\frac{1}{2}\right) - \sin(\alpha)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\sin(\alpha) + \cos(\alpha)\left(\frac{1}{2}\right) \\ &= \left(\frac{1}{2}\right)\cos(\alpha) + \left(\frac{1}{2}\right)\cos(\alpha) = \cos(\alpha) \end{aligned}$$

4. Provide a counterexample to show that $\sin(x) + 2\cos(x) = 0$ is not an identity.

$$\text{Let } x = \frac{\pi}{2}, \text{ then } \sin(x) + 2\cos(x) = \sin\left(\frac{\pi}{2}\right) + 2\cos\left(\frac{\pi}{2}\right) = 1 + 2 \neq 0$$

$\frac{\pi}{2}$ is a counterexample to the statement and therefore the statement is not an identity.