SLU PreCalculus TEST 3 Sections 6.2 – 6.5, 7.1, 7.2 Spring 2007

NAME: ______________________________ Score ________________________ /100

Please print

SHOW ALL YOUR WORK IN A NEAT AND ORGANIZED FASHION

You may use a calculator, but anytime you use the calculator your work must make it very clear to me what you did to arrive at your answer.

Each questions is worth 10 points.

1. Find the exact value of sin(105°). A calculator should not be used.

Consider 105° = 60° + 45° and use formula for the sin of a sum.

sin(105°) = sin(60° + 45°) = sin(60°)cos(45°) + sin(45°)cos(60°)

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

2. Prove/Verify $\sin(x - 45°) = \frac{\sin x - \cos x}{\sqrt{2}}$ (Example 14 on page 491)

Proof: sin(x - 45°) = sin(x)cos(45°) – sin(45°)cos(x)

$$= \sin(x)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}}\right)\cos(x) = \left(\frac{\sin(x) - \cos(x)}{\sqrt{2}}\right)$$

3. Find all exact solutions in [0, 2π] for the equation $2\cos^2(x) - \cos(x) = 0$. A calculator should not be used. (Example 1 on page 511)

$2\cos^2(x) - \cos(x) = 0$

cos(x)[2cos(x) – 1] = 0

cos(x) = 0 OR 2cos(x) – 1 = 0 which is equivalent to $\cos(x) = \frac{1}{2}$

The solutions of cos(x) = 0 in [0, 2π] are $\frac{\pi}{2}$ and $\frac{3\pi}{2}$	The solutions of $\cos(x) = \frac{1}{2}$ in [0, 2π] are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

4. Find <u>ALL</u> solutions of the equation $\sqrt{2}\sin(x) - 1 = 0$. A calculator should not be used. (Exer. 12 on page 519)

The equation $\sqrt{2}\sin(x) - 1 = 0$ is equivalent to $\sin(x) = \frac{1}{\sqrt{2}}$.

The solution of this equation in Quadrant I is $\frac{\pi}{4}$ and in Quadrant II is $\frac{3\pi}{4}$

Because the period of sin is 2π, the set of all solutions is $\frac{\pi}{4} + 2k\pi$ and $\frac{3\pi}{4} + 2k\pi$ for all integers k

5. Use a calculator to find a solution of 5.0118sin(x) – 3.1105 = 0 in Quadrant II (Exer. 18 on page 519)

The equation 5.0118sin(x) – 3.1105 = 0 is equivalent to $\sin(x) = \frac{3.1105}{5.0118}$

Then $x = \sin^{-1} \circ \sin(x) = \sin^{-1}\left(\frac{3.1105}{5.0118}\right) = 38.36°$ (0.6696 radians) is a solution in Quadrant I

The solution in Quadrant II is 180° - 38.36° = 141.64° (π - 0.6696 = 2.4720 radians) is the second Quadrant solution.

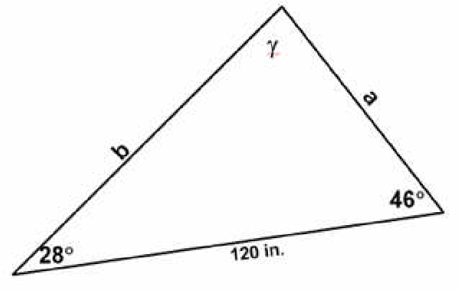
6. Solve Triangle 1. (Like Example 1 on page 532)

$\gamma = 180° - 28° - 46° = 106°$

Then the Law of Sines yields $\dfrac{\sin(106°)}{120} = \dfrac{\sin(46°)}{b}$ and $\dfrac{\sin(106°)}{120} = \dfrac{\sin(28°)}{a}$

So that $b = (120)\left(\dfrac{\sin(46°)}{\sin(106°)}\right) = 89.79$

and $a = (120)\left(\dfrac{\sin(28°)}{\sin(106°)}\right) = 58.6$



Triangle 1

7. Solve Triangle 2. (Like Matched Problem 1 on page 533)

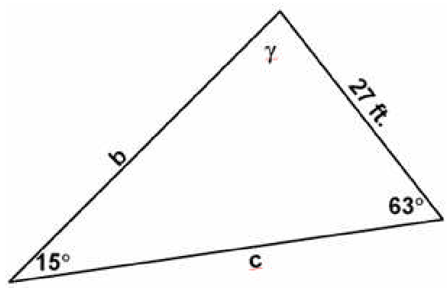
$\gamma = 180° - 63° - 15° = 102°$

Then the Law of Sines yields $\dfrac{\sin(106°)}{120} = \dfrac{\sin(46°)}{b}$ and

$\dfrac{\sin(106°)}{120} = \dfrac{\sin(28°)}{a}$

So that $b = (120)\left(\dfrac{\sin(46°)}{\sin(106°)}\right) = 89.79$

and $a = (120)\left(\dfrac{\sin(28°)}{\sin(106°)}\right) = 58.6$



Triangle 2

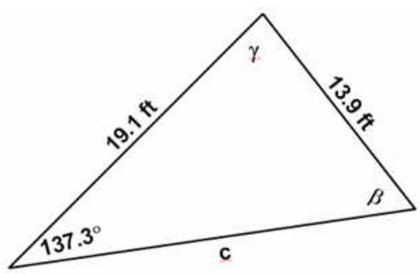
8. Solve Triangle 3. (Exercise 28 on page 539)

From the Law of Sines we obtain $\dfrac{\sin(\beta)}{19.1} = \dfrac{\sin(137.3°)}{13.9}$ so that

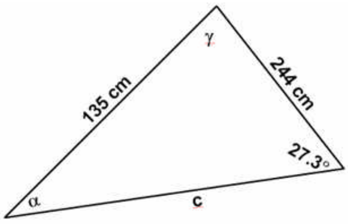
$\beta = \sin^{-1} \circ \sin(\beta) = \left(\dfrac{19.1}{13.9}\right)\sin(137.3°) = 68.7°$

However, $137.3° + 68.7° = 206° > 180°$

Therefore there is no solution.



Triangle 3



Triangle 4

9. Solve Triangle 4. (Exercise 26 on page 539)
Use the Law of Sines to determine α.

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b}$$

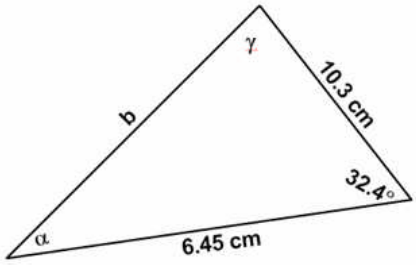
$$\frac{\sin(\alpha)}{a} = a\left(\frac{\sin(\beta)}{b}\right) = (244)\left(\frac{\sin(27.3°)}{135}\right) = 0.8290$$

$$\alpha = \sin^{-1}(0.829) = 56.0° \text{ in QI or } 124° \text{ in Q II}$$

Triangle I: α = 56°	**Triangle II: α = 124°**
γ = 180° - 56° - 27.3° = 96.7° $c = (135)\dfrac{\sin(96.7°)}{\sin(27.3°)} = 292$	γ = 180° - 124° - 27.3° = 28.7° $c = (135)\dfrac{\sin(28.7°)}{\sin(27.3°)} = 141$

10. Solve Triangle 5 (Example 1 on page 543)
Use the Law of Cosines in the form $b^2 = a^2 + c^2 - 2ac\cos(\beta)$ to solve for b.

$$b = \sqrt{a^2 + c^2 - 2ac\cos(\beta)}$$
$$= \sqrt{(10.3)^2 + (6.45)^2 - (2)(10.3)(6.45)\cos(32.4°)}$$
$$= 5.96$$



Triangle 5

Use the Law of Sines to solve for the **angle opposite the shorter side**. So we solve for γ using $\frac{\sin(\gamma)}{\gamma} = \frac{\sin(\beta)}{b}$

$$\sin(\gamma) = (c)\left(\frac{\sin(\beta)}{b}\right) = (6.45)\left(\frac{\sin(32.4)}{5.96}\right) = 0.5799$$

Then $\gamma = \sin^{-1}\circ\sin(\gamma) = \sin^{-1}(0.5799) = 35.4°$

And finally α = 180° - (β + γ) = 180° - (32.4° - 35.4°) = 112.2°

Note

$$\frac{\sin\gamma}{c} = \frac{\sin(35.4°)}{6.45} = 0.089 \qquad \frac{\sin\alpha}{a} = \frac{\sin(112.2°)}{10.3} = 0.089 \qquad \frac{\sin\beta}{b} = \frac{\sin(32.4°)}{5.96} = 0.089$$

validating the Law of Sines.

Notice that if the Law of Sines is used to find α before finding γ, then we obtain

$$\sin(\alpha) = (a)\left(\frac{\sin(\beta)}{b}\right) = (10.3)\left(\frac{\sin(32.4°)}{5.96}\right) = 0.9260$$

$$\alpha = \sin^{-1}(0.9260) = 67.8°$$

From which it follows that γ= 79.8°

Now note the following:

$$\frac{\sin\gamma}{c} = \frac{\sin(78.8°)}{6.45} = 0.153 \qquad \frac{\sin\alpha}{a} = \frac{\sin(67.8°)}{10.3} = 0.089 \qquad \frac{\sin\beta}{b} = \frac{\sin(32.4°)}{5.96} = 0.089$$

violating the Law of Sines! Therefore those values for α and γ are branded as incorrect. However, note that 180° - α = 180° - 67.8° = 112.2° which is the value of α found in the correct solution and then of course γ = 35.4°.