TRIG CHEAT SHEET

360 degrees is equal to 2π radians.

Derive both conversion formulas from $360^\circ = 2\pi$ radians

Degree Measure	**Arc Length Radian Measure**	**Coordinates**	**Cos θ**	**Sin θ**
0	**0**	$\left(\frac{\sqrt{4}}{2}, \frac{\sqrt{0}}{2}\right) = (1,0)$	**1**	**0**
30°	**π/6**	$\left(\frac{\sqrt{3}}{2}, \frac{\sqrt{1}}{2}\right) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
45°	**π/4**	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
60°	**π/3**	$\left(\frac{\sqrt{1}}{2}, \frac{\sqrt{3}}{2}\right) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
90°	**π/2**	$\left(\frac{\sqrt{0}}{2}, \frac{\sqrt{4}}{2}\right) = \mathbf{(0,1)}$	**0**	**1**

For each real number x, the wrapping function W associates a point on the unit circle. The sine and cosine functions are defined as follows:

$\sin(x)$ is the second coordinate of the point on the unit circle .

$\cos(x)$ is the second coordinate of the point on the unit circle.

The other six trig functions may be defined by what are usually referred to as the reciprocal identities or quotient identities.

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

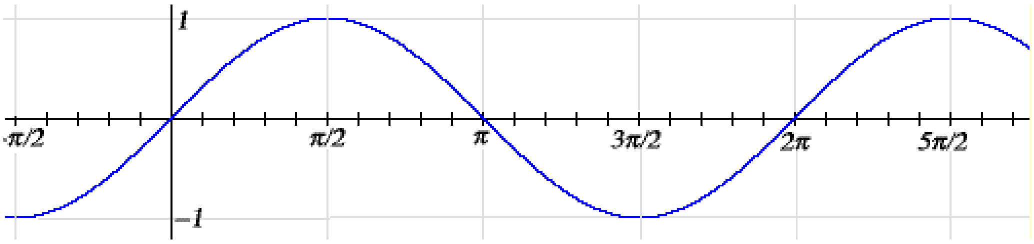
The squared, or Pythagorean Identities are consequences of the equation $x^2 + y^2 = 1$ of the unit circle and the definition of sin and cos.

$$\sin^2(x) + \cos^2(x) = 1 \qquad \tan^2(x) + 1 = \sec^2(x) \qquad 1 + \cot^2(x) = \csc^2(x)$$

Identities for negatives follow from the fact that W(x) and W(–x) are symmetrical with respect to the x –axis.

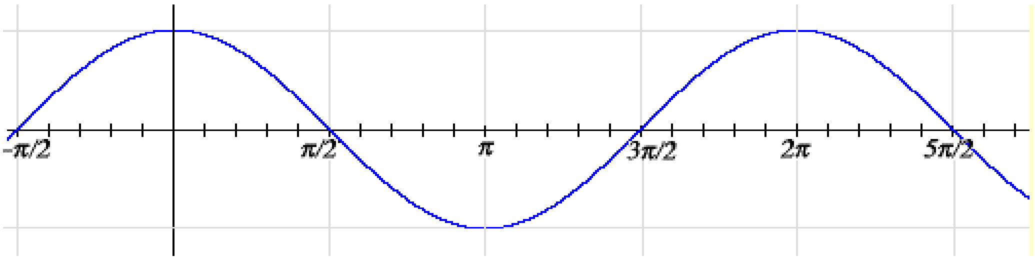
$$\sin(-x) = -\sin(x) \qquad \cos(-x) = \cos(x) \qquad \tan(-x) = -\tan(x)$$

The Sine Function:



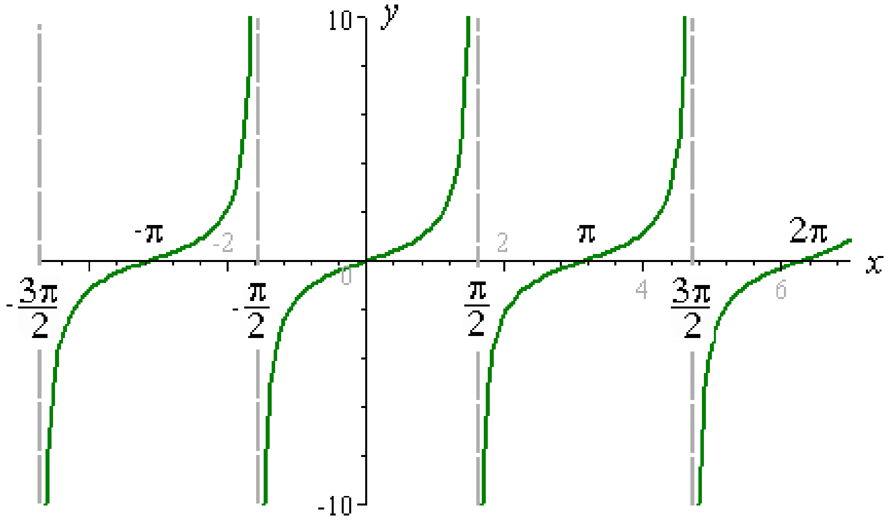
The domain of sin is all real numbers.
The range of sin is [-1, 1]
The zeros of sin are the multiples of π
The sin function is periodic with period 2π
The sin function is positive in quadrants I and II and negative in quadrants III and IV
The sin function is not one-to-one (does not pass the horizontal line test) and therefore has no inverse.
The sin function with its domain restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is one_to_one and has an inverse $\sin^{-1}$
Another symbol used for the inverse of sin is arcsin

The Cosine Function:



The domain of cos is all real numbers.
The range of cos is [-1, 1]
The zeros of cos are odd multiples of $\frac{\pi}{2}$
The cos function is periodic with period 2π
The cos function is positive in quadrants I and IV and negative in quadrants II and III
The cos function is not one-to-one (does not pass the horizontal line test) and therefore has no inverse.
The cos function with its domain restricted to $[0, \pi]$ is one_to_one and has an inverse $\cos^{-1}$
Another symbol used for the inverse of cos is arccos

The Tangent Function:



Recall that the domain of a rational function is all real numbers except the zeros of its denominator. Since the denominator of the tangent function is the cosine function, the domain of tan is all real numbers except the zeros of the cos function. The domain of tan is all real numbers except odd multiples of $\frac{\pi}{2}$.

Furthermore tan has vertical asymptotes at the odd multiples of $\frac{\pi}{2}$.

The range of tan is all real numbers.
The tan function is periodic with period π
The tan function is positive in quadrants I and III and negative in quadrants II and IV
The tan function is increasing everywhere it is defined.
The tan function is not one-to-one (does not pass the horizontal line test) and therefore has no inverse.
The tan function with its domain restricted to $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ is one_to_one and has an inverse $\cos^{-1}$

Triangle Trig:

$\sin(\theta) = \dfrac{opp}{hyp}$ $\cos(\theta) = \dfrac{adj}{hyp}$ $\tan(\theta) = \dfrac{opp}{adj}$

Where θ may be either α or β

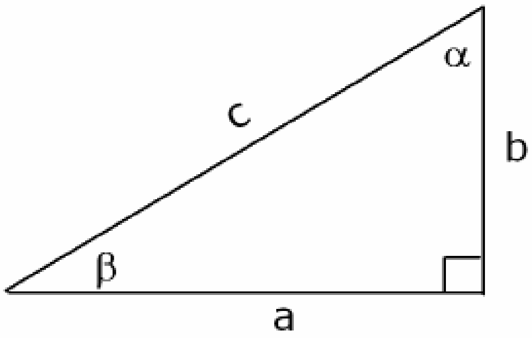


Figure 1

Cofunction Identities:

Definition: Two angles are complementary angles if their sum is 90°.

Property: The sum of the interior angles of a triangle is $180°$.

Property: The two acute angles in a right triangle are complementary angles.

Refer to Figure 1 to observe the following cofunction identities.

If α and β are complementary angles, then

$\sin(\beta) = \cos(\alpha)$
$\tan(\beta) = \cot(\alpha)$
$\sec(\beta) = \csc(\alpha)$

The cofunction of an angle equals the function of the complementary angle

If β is an angle, then its complement is $\frac{\pi}{2} - \beta$. This gives rise to an alternate statement of the so-called cofunction identities.

$$\sin(\beta) = \cos(\frac{\pi}{2} - \beta)$$

$$\tan(\beta) = \cot(\frac{\pi}{2} - \beta)$$

$$\sec(\beta) = \csc(\frac{\pi}{2} - \beta)$$

Amplitude, Period, and Phase Shift:

Let A, B and C be real number constants with $A \neq 0$ and $B > 0$.

For $f(x) = A\sin(Bx + C)$ and $f(x) = A\cos(Bx + C)$

Amplitude = $|A|$ Period = $\dfrac{2\pi}{B}$ Phase Shift = $\dfrac{-C}{B}$

For $f(x) = A\tan(Bx + C)$

Period = $\dfrac{\pi}{B}$ Phase Shift = $\dfrac{-C}{B}$

Sum and Difference Formulas:

cos (x – y) = cos(x)cos(y) + sin(x)sin(y)
because x – y = x + (–y), and
because cos(–y) = cos(y) and
because sin(–y) = –sin(y)
cos (x + y) = cos(x)cos(y) – sin(x)sin(y)

These two identities and the cofunction identities permit a development of identities for sum and difference of the sine and tangent functions.

sin(x – y) = sin(x)cos(y) – cos(x)sin(y)
sin(x + y) = sin(x)cos(y) + cos(x)sin(y)

$$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$$

$$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$$

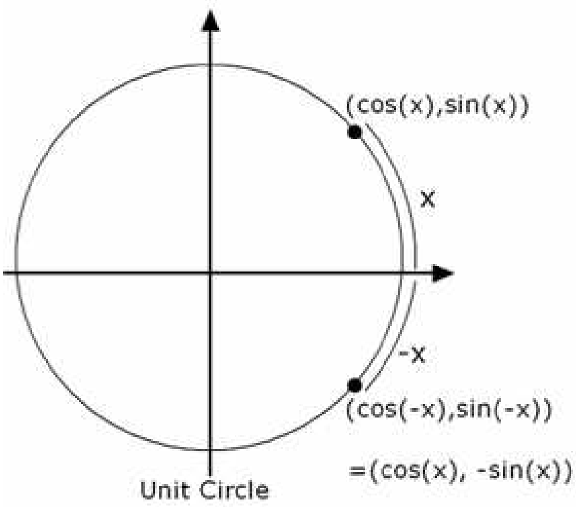


Figure 2

Double-Angle Identities

sin(2x) = 2 sin(x)cos(x)
$\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$

$$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)} = \frac{2\cot(x)}{\cot^2(x)-1} = \frac{2}{\cot(x)-\tan(x)}$$

From the three identities for cos(2x) we can derive

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

Half-Angle Identities