

Mixture Problem – Explanation

There is a difference between percent and amount. Percent is a rate. Amount is a quantity. If I told you I drove 60 mph for 3 hours and asked how far I had driven, you would not respond 60 mph (a rate) but rather you would state the quantity 180 miles. It is important that you make the distinction between rate and amount, particularly throughout the process of solving a mixture problem. When analyzing mixture problems it is essential to convert to and discuss quantities (**amounts**) not rates.

When analyzing a mixture problem, as in many other problems, a winning strategy is to find some quantity (**amount**) in the problem which can be expressed in two different ways thus arriving at an equation which can be solved.

Consider the following mixture problem.

Problem: What quantity (**amount**) of a 60% acid solution must be mixed with a 30% acid solution to produce 300 mL of a 50% acid solution?

Analysis: In this analysis (an integral part of the solution process) I will state the steps in logical order. Beneath each step, in small blue indented type, I will state the reason/logic/explanation for that step.

(1) Let x be the **amount** of the 60% solution to be added.

Begin by assigning a variable to the quantity (not the rate) of material you are to determine.

(2) The **amount** of the final solution is 300 mL.

The problem statement dictates that the final quantity be 300 mL.

(3) The **amount** of acid in the **final solution** is $(0.5)(300)$

The problem statement dictates that the final solution be half acid (a 50% solution).

Observe that $(0.5)(300)$ is one way of describing the amount of acid in the final solution. Our strategy is to find another way of describing the amount of acid in the final solution. Those two ways of describing the amount of acid in the final solution will produce an equation. That equation is the mathematical model for this mixture question. Note that we are **focusing on the amount of acid in the final solution.**

Now try to write the amount of acid in the final solution by calculating the amount contributed by each of the addends

(4) The **amount** of acid contributed by the 60% solution is $0.6x$

This is the meaning of a 60% solution. If 60% of x mL is acid then the amount of acid is $.6x$ mL.

(5) The **amount** of 30% solution is $300 - x$

The sum of the amount of 60% solution and amount of 30% solution is 300. There are x liters of 60% so there must be $300 - x$ liters of 30% solution

(6) The **amount** of acid contributed by the 30% solution is $(0.3)(300 - x)$

This is the meaning of a 30% solution. If 30% of $(300 - x)$ mL is acid then the amount of acid is $(0.3)(300 - x)$ mL.

(7) The **amount** of acid in the **final solution** is therefore

$$0.6x + (0.3)(300 - x).$$

This is the sum of the amount contributed by the 60% solution (Statement 4) and the amount contributed by the 30% solution (Statement 6).

(8) The model for this mixture question is $0.6x + (0.3)(300 - x) = (0.5)(300)$

This equation results from Statement 3 and Statement 7.

Solving the Equation:

(9) Ordinary processes are used to solve this equation as illustrated below.

$$0.6x + (0.3)(300 - x) = (0.5)(300)$$

Multiply both sides by 10 to clear decimals.

$$6x + 3(300 - x) = (5)(300)$$

Expand each of the expression

$$6x + 900 - 3x = 1500$$

Add - 900 to both sides and add like terms

$$3x = 600$$

Multiply both sides by 1/3

$$x = 200$$

Conclusion:

(10) 200 mL of 60% acid solution must be added to 100 mL of 30% acid solution to produce 300 mL of 50% acid solution.